THE HOUSTON COMMUNITY COLLEGE - SOUTHWEST

MATH 1314 FINAL REVIEW PROBLEMS

11-08-11

These exercises represent a compilation of typical problems in this course. This is NOT a sample of the final exam. However, doing these problems will help you prepare for the final exam

1. Solve the equation $(9x+5)^2 = 2$.

2. Solve the equation

   $3x^2 + 6x = -4$

3. Use algebraic tests to check the following for symmetry with respect to the axes and the origin.
   $y = 5x^5 - x^3 + 1$

4. Write the standard form of the equation of the circle with the given characteristics.
   endpoints of a diameter: $(-1,\ 4)$, and $(7,\ 6)$

5. Solve the inequality.

   $9x^2 + 24x > -16$

6. Solve the inequality.

   $\dfrac{x-1}{x+5} \geq 0$

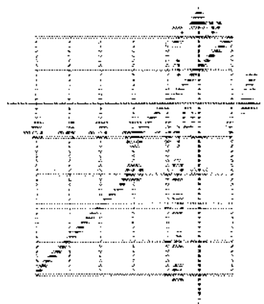
7. Solve the inequality. Express the solution set in interval notation.

   $|\,2x - 1\,| - 9 > 2$

8. Write the slope-intercept form of the equation of the line through the given point perpendicular to the given line.

   point: $(-8,\ 8)$ line: $-5x - 15y = 5$

9. Use the graph of the function to find the domain and range of f.



10. Find all solutions to the following equation.

$x - \sqrt{2x-4} = 2$

11. Find the domain of the function.

$y = \sqrt{1-2x}$

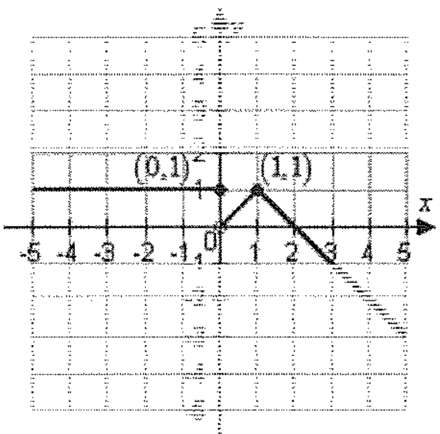
12. Evaluate the function at the specified value of the independent variable and simplify.

$$f(t) = \begin{cases} t, & t \le -1 \\ t^2 - 3t, & -1 \le t \le 1 \\ t^3 - 3t^2, & t > 1 \end{cases}$$

$f\left(\frac{1}{3}\right)$

13. Determine the intervals over which the function is increasing, decreasing, or constant.

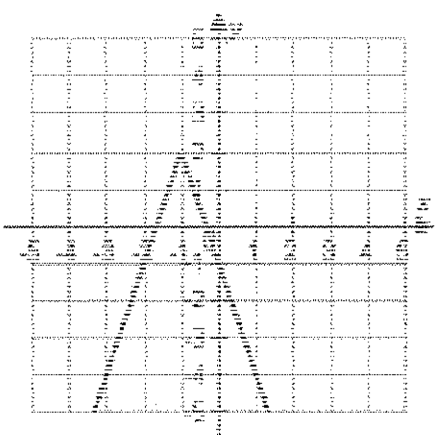
$$f(x) = \begin{cases} 1, & x < 1 \\ -|x - 1| + 1, & x \geq 1 \end{cases}$$



14. Use the graph of

$f(x) = |x|$

to write an equation for the function whose graph is shown.



15. Evaluate the indicated function for $f(x) = x^2 + 9$ and $g(x) = x - 7$.

$(f - g)(t - 9)$

16. Evaluate the indicated function for $f(x) = x^2 - 7$ and $g(x) = x - 8$.

$(fg)(-1)$

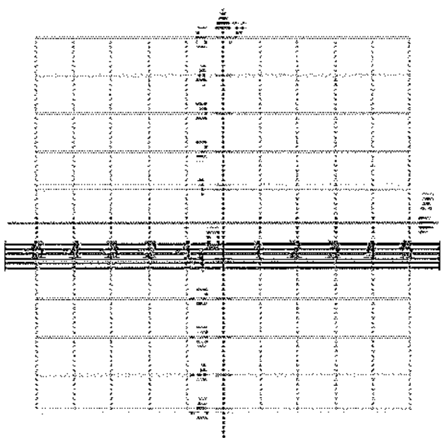
17. Find $g \circ f$.

$f(x) = x - 1 \qquad g(x) = x^2 + x$

18. Describe the right-hand and the left-hand behavior of the graph of $n(x) = -\frac{8}{11}\left(x^3 - 4x^2 + x + 1\right)$.

19. Graph the given function.

$f(x) = (x - 2)^2 - 1$



20. Find the inverse function of f.

$f(x) = \frac{3x-4}{4x-7},\ x \neq \frac{7}{4}$

21. Determine whether the function has an inverse function. If it does, find the inverse function.

$f(x) = (x-4)^3 + 6$

22. From the graph of the quadratic function $f(x) = -3x^2 + 6x + 6$, determine the equation of the axis of symmetry.

23. Write the quadratic function $f(x) = -x^2 + 16x - 61$ in standard form.

24. Identify the center and radius of the circle.

$(x+5)^2 + (y+4)^2 = 64$

25. Find all real zeros of the polynomial $f(x) = x^4 + 10x^3 + 9x^2$ and determine the multiplicity of each.

26. Use synthetic division to divide.

$\left(x^3 - 75x + 250\right) \div (x - 5)$

27. Given that one of the factors is $(x+3)$ find the remaining factor(s) of $f(x) = x^3 + 9x^2 + 26x + 24$ and write the polynomial in fully factored form.

28. Write all <u>possible rational zeros</u> of the function $f(x) = -2x^4 + 4x^3 + 79x^2 - 100x + 10.$, as per rational zeros theorem.

29. Determine all zeros of $f(x) = x^3 - 3x^2 - 16x + 48$.

30. Determine the equations of any horizontal and vertical asymptotes of $f(x) = \dfrac{x^2 - 4}{x^2 + x - 6}$.

31. Find the vertical and horizontal asymptotes of $f(x) = \dfrac{x - 9}{x^2 - 81}$.

32. Find the domain of $f(x) = \dfrac{x^2 - 36}{x^2 + x - 42}$.

33. Find the exact value of $\log_7 \sqrt[3]{49}$ without using a calculator.

34. Rewrite the logarithm $\log_3 142$ in terms of the common logarithm .

35. Use properties of logarithm to expand the expression as a sum, difference, and/ or constant multiple of logarithms

$\log \dfrac{(x-1)^3}{y^2 z}$

36. Condense the expression $7(\log x - \log y) + 2 \log z$ to the logarithm of a single term.

37. Solve the system

$$\begin{cases} x - y = -1 \\ x^2 - y = 5 \end{cases}$$

38. Solve the system $\begin{cases} \frac{7}{9}x + \frac{1}{9}y = \frac{8}{9} \\ 7x + y = 8 \end{cases}$

39. Solve using any method.

$$\begin{cases} 2x + 9y = -9 \\ 9x - 8y = -19 \end{cases}$$

40. Rewrite the exponential equation $5^{-3} = \frac{1}{125}$ in logarithmic form.

41. Solve the equation..

$2e^{x+3} = 5$

42. Solve the equation.

$\log x + \log(x-3) = 1$

43. Solve for X in the equation given.

$-2X = 4A - B$, $A = \begin{bmatrix} -2 & -3 \\ -1 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -16 \\ 6 & -42 \end{bmatrix}$

44. Find the determinant of the matrix $\begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -5 & -\frac{1}{3} \end{bmatrix}$.

45. If possible, find AB.

$A = \begin{bmatrix} 8 & -4 \\ -6 & 1 \\ 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$

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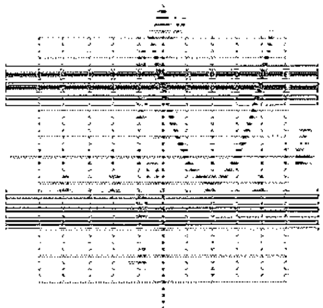
Answer Section

SHORT ANSWER

1. $x = \dfrac{-5+\sqrt{2}}{9}, \quad \dfrac{-5-\sqrt{2}}{9}$
2. $x = \dfrac{-3+\sqrt{3}\,i}{3}, \dfrac{-3-\sqrt{3}\,i}{3}$
3. no symmetry
4. $(x-3)^2 + (y-5)^2 = 17$
5. $\left(-\infty, -\dfrac{4}{3}\right) \cup \left(-\dfrac{4}{3}, \infty\right)$
6. $(-\infty, -5) \cup [1, \infty)$
7. $(-\infty, -5) \cup (6, \infty)$
8. $y = 3x + 32$
9. domain: $(-\infty, -2) \cup (-2, \infty)$
   range: $(-\infty, -2) \cup (-1, \infty)$
10. $x = 4, \quad x = 2$
11. $(-\infty, \dfrac{1}{2}]$
12. $-\dfrac{8}{9}$
13. constant on $(-\infty, 0)$

    increasing on $(0, 1)$

    decreasing on $(1, \infty)$
14. $f(x) = -3|x+1| + 2$
15. $t^2 - 19t + 106$
16. 54
17. $(g \circ f)(x) = x^2 - x$
18. Because the degree is odd and the leading coefficient is negative, the graph rises to the left and falls to the right.

19. 
20. $f^{-1}(x) = \dfrac{7x-4}{4x-3}, x \neq \dfrac{3}{4}$
21. $f^{-1}(x) = \sqrt[3]{x-6}+4$
22. $x = 1$
23. $f(x) = -(x-8)^2+3$
24. center: $(-5, -4)$ radius: 8
25. $x = 0$, multiplicity 2; $x = -9$, multiplicity 1; $x = -1$, multiplicity 1
26. $x^2+5x-50$
27. $f(x) = (x+3)(x+4)(x+2)$
28. $x = 1,-1,2,-2,5,-5,10,-10,\frac{1}{2},-\frac{1}{2},\frac{5}{2},-\frac{5}{2}$
29. x=3, 4,-4
30. horizontal: $y = 1$; vertical: $x = -3$
31. Vertical asymptote x = -9, Horizintal asymptote : y =0
32. all real numbers except $x = 6$ and $x = -7$
33. $\frac{2}{3}$
34. $\dfrac{\log 142}{\log 3}$
35. $3\log(x-1)-2\log y-\log z$
36. $\log\left(\frac{x}{y}\right)^7 z^2$
37. $(-2,-1), (3, 4)$
38. $(a, 8-7a)$ (dependent)
39. $\left(-\frac{243}{97}, -\frac{43}{97}\right)$
40. $\log_5 \frac{1}{125} = -3$
41. $x = \ln\left(\frac{5}{2}\right) - 3$
42. $x = 5$

-873
387
$\overline{486}$

43. $\begin{bmatrix} 3 & -2 \\ 5 & -5 \end{bmatrix}$

44. $-\dfrac{65}{9}$

45. $\begin{bmatrix} -20 \\ 21 \\ -30 \end{bmatrix}$

Detailed Answers To "Math 1314 Review Problems"

1- Solve the equation:

$$(9x+5)^2 = 2$$

- Solution -

Take the square root of both sides of the equation:

$$\sqrt{(9x+5)^2} = \sqrt{2}$$

you get

$$9x+5 = \pm\sqrt{2}$$

Now solve for x.

$$9x = -5 \pm \sqrt{2}$$

Divide both sides of the equation by 9.

$$x = \frac{-5 \pm \sqrt{2}}{9} \quad \text{(answer)}$$

2- Solve the equation:

$$3x^2 + 6x = -4$$

- Solution -

This is a Quadratic Equation.

you have to make the equation $= 0$ first

$$3x^2 + 6x + 4 = 0$$

The Quadratic Formula is

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3, \quad b = 6, \quad c = 4$

First find $b^2 - 4ac$

$= (6)^2 - 4(3)(4)$

$= 36 - 48 = -12$

Now replace the answer in the formula:

$$X = \frac{-6 \pm \sqrt{-12}}{6}$$

Let's simplify $\sqrt{-12}$ first:

$\sqrt{-12} = \sqrt{-1 \times 12}$, but $i^2 = -1$

Therefore $\sqrt{-12} = \sqrt{12i^2}$. Rewrite 12 as 4×3.

$\sqrt{4 \times 3 \times i^2} = 2i\sqrt{3}$.

therefore $X = \dfrac{-6 \pm 2i\sqrt{3}}{6}$

Finally divide each term by 2.

$X = \dfrac{-3 \pm i\sqrt{3}}{3}$. (answer)

3. Use algebraic tests to check for symmetry:

$y = 5x^5 - x^3 + 1$

- solution -

To be symmetric with respect to the origin:

$f(-x) = -f(x)$

Let's pick any negative number. Let's say -1.

$f(-1) = 5(-1)^5 - (-1)^3 + 1$

$= -5 + 1 + 1 = -3$

$f(1) = 5(1)^5 - (1)^3 + 1$

$= 5 - 1 + 1 = 5$

Is $f(-1) = -f(1)$?

Is $-3 = -5$? No

Therefore it is not symmetric with respect to the origin.

It is also not symmetric with respect to the x or y axis since the power of y and x are not even.

Answer: (NO SYMMETRY)

4- Write the standard form of the equation of the circle with the given characteristics.
End points of a diameter: $(-1, 4)$, and $(7, 6)$

- Solution -

Center is the mid point of the diameter.

$$x_{Mid\,pt} = \frac{-1+7}{2} = 3 \;;\; y_{Mid\,pt} = \frac{4+6}{2} = 5$$

Center $(3, 5)$.

Radius is from the center to any of the end point of the diameter.

Distance from: $(\underset{x_1}{3}, \underset{y_1}{5})$ to $(\underset{x_2}{7}, \underset{y_2}{6})$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7-3)^2 + (6-5)^2} = \sqrt{16+1} = \sqrt{17}.$$

Equation of any circle:

$$(x-a)^2 + (y-b)^2 = R^2$$

$$(x-3)^2 + (y-5)^2 = 17$$

(5)

5, Solve the inequality:

$$9x^2 + 24x > -16$$

– Solution –

Make the inequality > 0 by moving -16 to the other side.

$$9x^2 + 24x + 16 > 0$$

Factor $9x^2 + 24x + 16$:

$$(3x + 4)(3x + 4)$$

Set the factors to 0 and solve for x.

$$3x + 4 = 0$$
$$\underline{\quad -4 \quad -4}$$
$$3x = -4$$
$$x = -4/3.$$

Now pick a value for $x > -\frac{4}{3}$ like 0 and substitute it in $(3x + 4)(3x + 4)$, you get

$$(4)(4) = 16$$

and it is > 0.

Therefore any number for $x > \frac{-4}{3}$ will make $9x^2 + 24x + 16 > 0$.

Now pick x to be smaller than or $< -\frac{4}{3}$ like -2. ~~+~~ substitute it in

$$(3x + 4)(3x + 4)$$
$$(-6 + 4)(-6 + 4)$$
$$= (-2)(-2)$$

It is also positive or > 0. Therefore when $x < -4/3$ or $x > -\frac{4}{3}$

$$9x^2 + 24x + 16 > 0.$$

6, Solve the inequality:

$$\frac{(x-1)}{(x+5)} \geq 0$$

\- Solution -

Rewrite the Problem as:

$$(x-1)(x+5) \geq 0$$

First solve the equation:

$$(x-1)(x+5) = 0$$

$$x = 1 \quad \text{or } x = -5$$

Pick a number for x between -5 and 1.

Let's say 0.

Replace 0 in

$$(x-1)(x+5)$$

$$(0-1)(0+5) = -5$$

Therefore if x is between -5 and 1

$(x-1)(x+5)$ is < 0.

Pick x now Either > 1 or < -5

like 2.

$$(x-1)(x+5)$$

$$(2-1)(2+5) = 7 \text{ which is } > 0$$

Therefore $\frac{(x-1)}{(x+5)}$ is > 0 when

$x < -5$ or $x > 1$ and $= 0$ when

$x = 1$.

Answer is : $(-\infty, -5) \cup (1, \infty)$

7. Solve the inequality:

$$|2x-1| - 9 > 2$$

- Solution -

Isolate the Absolute Value term first.

$$|2x-1| > 11$$

<u>Rule is:</u> If $|x| > a$ then

$$x > a$$

or

$$x < -a$$

Therefore:

$$\begin{array}{r} 2x - 1 > 11 \\ +1 \quad +1 \\ \hline 2x > 12 \end{array}$$

$$x > 6$$

$$\begin{array}{r} 2x - 1 < -11 \\ +1 \quad +1 \\ \hline 2x < -10 \end{array}$$

$$x < -5$$

Answer is: $(-\infty, -5) \cup (6, \infty)$

8, Write the slope-intercept form of the equation of the line through the given point perpendicular to the given line.

point: $(-8, 8)$ $\quad$ line: $-5x - 15y = 5$

$x_1 \;\; y_1$

- Solution -

Equation of any line is $y = mx + b$.

Since the lines are perpendicular, the product of the slopes should $= -1$.

Let's find the slope of: $-5\cancel{y} - 15y = 5$.

Get "y" by itself:

$$-15y = 5x + 5$$

Divide by -15

$$y = \frac{5}{-15}x + \frac{5}{-15}$$

$$y = -\frac{1}{3}x - \frac{1}{3}$$

; slope $= -\frac{1}{3}$. slope of the line perpendicular to it is 3.

Equation of any line in point-slope form is:

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 3(x + 8)$$

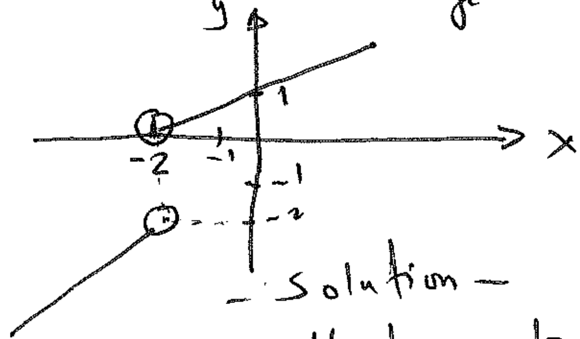
$$y - 8 = 3x + 24$$

$$+8 \qquad\qquad + 8$$

$$y = 3x + 32$$

answer

9- Find the Domain and Range.



– Solution –

Domain $\Longrightarrow$ Smallest x to largest x : $(-\infty, -2) \cup (-2, \infty)$

Range $\Longrightarrow$ Smallest y to largest y

$(-\infty, -2) \cup (0, \infty)$

10- Solve:

$x - \sqrt{2x-4} = 2$

solution

Isolate the radical by moving x to the other side.

$-\sqrt{2x-4} = 2 - x$

Multiply both Equations by (-1)

$\sqrt{2x-4} = -2 + x$

SQuare both sides of the Equation

$2x - 4 = x^2 - 4x + 4$

Set the Equation to 0.

$x^2 - 4x + 4 - 2x + 4 = 0$

$x^2 - 6x + 8 = 0$

Factor

$(x-2)(x-4) = 0$

$x = 2$ or $x = 4$

11. Find the domain of the function.

$y = \sqrt{1-2x}$

– solution –

Set $\sqrt{1-2x} \geq 0$ and solve for x

$$1 - 2x \geq 0$$

$$\underline{-1 \qquad\quad -1}$$

$$-2x \geq -1$$

$$x \leq \frac{1}{2} \quad \text{or} \quad (-\infty, \tfrac{1}{2}]$$

12, Evaluate the function at the specified value of the independent variable and simplify.

$$f(t) = \begin{cases} t, & t \leq -1 \\ t^2 - 3t, & -1 \leq t \leq 1 \\ t^3 - 3t^2, & t > 1 \end{cases}$$

find $f(\frac{1}{3})$.

Solution

The domain $\frac{1}{3}$ is defined in $t^2 - 3t$

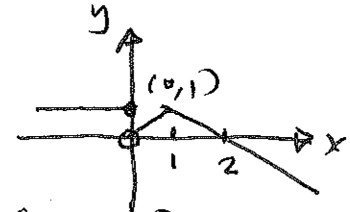
Replace t with $\frac{1}{3}$

$$\left(\tfrac{1}{3}\right)^2 - 3\left(\tfrac{1}{3}\right) = \frac{1}{9} - 1 = \frac{1}{9} - \frac{9}{9}$$

$$= \frac{-8}{9}$$

13- Determine the intervals over which the function is increasing, decreasing, or constant.

$$f(x) = \begin{cases} 1, & x < 1 \\ -|x-1|+1 ; & x \geq 1 \end{cases}$$



- Solution -

Constant means (no ups or downs). $(-\infty, 0)$

increasing means (going upward). $(0, 1)$

Decreasing means (going downward). $(1, \infty)$.

14,

- Solution -

$f(x) = -3|x+1| + 2$

15, Evaluate the indicated function for $f(x) = x^2 + 9$ and $g(x) = x - 7$

$(f-g)(t-9)$

- Solution -

Find $f-g$ first

$x^2 + 9 - x + 7 = x^2 - x + 16$

Now replace x with $t-9$

$(t-9)^2 - (t-9) + 16$

$t^2 - 18t + 81 - t + 9 + 16$

Combine like Terms

$t^2 - 19t + 106$

16, Evaluate the indicated function for

$f(x) = x^2 - 7$

and

$g(x) = x - 8$

find $(fg)(-1)$

– Solution –

Find $f \cdot g$ first.

$(x^2 - 7)(x - 8) = x^3 - 8x^2 - 7x + 56$

Now replace x with -1

$(-1)^3 - 8(-1)^2 - 7(-1) + 56$

$= -1 - 8 + 7 + 56 = 54$

17, Find $g \circ f$

$f(x) = x - 1 \qquad g(x) = x^2 + x$

– Solution –

f is the domain and g is the range.

So replace $x - 1$ everywhere you see x

in $x^2 + x$.

$(x - 1)^2 + (x - 1)$

$= x^2 - 2x + 1 + x - 1 = x^2 - x$

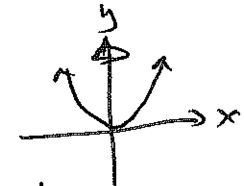
18, Describe the right-hand and left hand behavior of

$n(x) = \frac{-8}{11}(x^3 - 4x^2 + x + 1)$

- Solution -

Because the degree is odd and the leading coeficient is (-), the graph rises to the left and falls to the right.

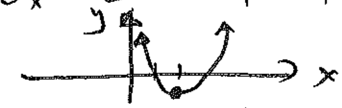
19, Graph the given function. $f(x) = (x-2)^2 - 1$



- Solution -

It is a parent function of $f(x) = x^2$

shift the vertex 2 units to the right and 1 unit down.



20, Find the inverse function of f.

$f(x) = \frac{3x-4}{4x-7} \qquad x \neq \frac{7}{4}$.

- Solution -

f(x) is the same as y.

Replace x with y and y with x.

$x = \frac{3y-4}{4y-7}$

cross multiply

$4xy - 7x = 3y - 4$

solve for y by bringing all the y terms to the same side.

$4xy - 7x - 3y = -4$

$4xy - 3y = -4 + 7x$

Take y as a common factor

$y(4x-3) = -4 + 7x$

$y = \frac{-4+7x}{4x-3} = f^{-1}$

21,

Determine whether the function has an inverse function. If it does, find the inverse function.

$f(x) = (x-4)^3 + 6$

- Solution -

Replace y with x and x with y

$x = (y-4)^3 + 6$

isolate $(y-4)^3$

$(y-4)^3 = x - 6$

cube root both sides

$\sqrt[3]{(y-4)^3} = \sqrt[3]{x-6}$

$y - 4 = \sqrt[3]{x-6}$

$y = \sqrt[3]{x-6} + 4 = f^{-1}$

22, From the graph of the quadratic function

$f(x) = -3x^2 + 6x + 6$

determine the equation of the axis of symmetry

- Solution -

axis of symmetry is $x = \frac{-b}{2a}$

$a = -3, \quad b = 6$

$x = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1$

23, Write the quadratic function
$f(x) = -x^2 + 16x - 61$ in standard form.

-Solution-

$f(x) = -x^2 + 16x - 61$

$= -(x^2 - 16x) - 61$. Make perfect squares.

$= -(x-8)^2 + 64 - 61$

$= -(x-8)^2 + 3$

24, Identify the center and radius of the circle.

$(x+5)^2 + (y+4)^2 = 64$

-Solution-

Equation of a circle is:

$(x-a)^2 + (y-b)^2 = R^2$.

center $(-5, -4)$. radius $= \sqrt{64} = 8$

26. Use synthetic division to divide.

$x^3 - 75x + 250 \div x - 5$

– Solution –

$$
\begin{array}{r|rrrr}
5 & 1 & 0 & -75 & 250 \\
 & & 5 & 25 & -250 \\
\hline
 & 1 & 5 & -50 & 0
\end{array}
$$

Answer is $x^2 + 5x - 50$

27, Given that one of the factors is $(x+3)$
find the remaining factors of
$f(x) = x^3 + 9x^2 + 26x + 24$
and write the Polynomial in fully factored form.

– Solution –

Since $x+3$ is a factor, therefore -3 is a zero. Divide by -3.

$$
\begin{array}{r|rrrr}
-3 & 1 & 9 & 26 & 24 \\
 & & -3 & -18 & -24 \\
\hline
 & 1 & 6 & 8 & 0
\end{array}
$$

The answer to the division is: ~~set it to 0~~
$x^2 + 6x + 8$
~~and solve for~~ factor it
$(x+2)(x+4)$
Final answer is $(x+3)(x+2)(x+4)$

28. Write all possible rational zeros of the function $f(x) = -2x^4 + 4x^3 + 79x^2 - 100x + 10$

-Solution-

list all factors of 10:

$\pm 10, \pm 5, \pm 2, \pm 1$ List A

list all factors of 2:

$\pm 2, \pm 1$ List B

Divide each number from list A by list B. (Do not duplicate answers)

$\frac{\pm 10}{2}, \frac{\pm 10}{1}, \frac{\pm 5}{2}, \frac{\pm 5}{1}, \frac{\pm 2}{2}, \frac{\pm 2}{1}$

$\frac{\pm 1}{2}, \frac{\pm 1}{1}$

$= \boxed{\pm 5, \pm 10, \pm 5/2, \pm 1, \pm 2, \pm 1/2}$

29, Determine all Zeros of:

$$f(x) = x^3 - 3x^2 - 16x + 48$$

– solution –

By listing possible rational Zeros:

$\pm 48, \pm 24, \pm 12, \pm 6, \pm 3, \pm 8, \pm 2, \pm 1$

$\pm 3, \pm 4$

Try 3 as a Zero.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & -16 & 48 \\ & & 3 & 0 & -48 \\ \hline & 1 & 0 & -16 & 0 \end{array}$$

Answer is $x^2 - 16$. Set it to 0.

$x = \pm 4$

30, Determine the equations of any horizontal and Vertical asymptotes of:

$$f(x) = \frac{x^2 - 4}{x^2 + x - 6}$$

– Solution –

Horizonal asymptote: = Coeficient of x^2 in the numerator divided by coeficient of x^2 in the denominator

$= \frac{1}{1} = 1$ $\boxed{y = 1}$.

Vertical asymptotes: set the denominator to 0. $x^2 + x - 6 = 0$; $(x+3)(x-2) = 0$

$x = -3$ and $x = 2$

31, Find the Vertical and Horizontal asymptotes of $f(x) = \dfrac{x-9}{x^2-81}$

\- Solution -

For Horizontal asymptote. Since the power of the numerator is smaller than the power in the denominator, then $\boxed{y=0}$.

For Vertical asymptote: Set the denominator to 0.

$$x^2 - 81 = 0$$
$$(x+8)(x-8) = 0$$
$$x = -8 \quad \text{and} \quad x = 8.$$

32, Find the domain of $f(x) = \dfrac{x^2-36}{x^2+x-42}$

\- Solution -

The domain is for all the Values of x except what makes the denominator 0.

$$x^2 + x - 42 = 0$$
$$(x+7)(x-6) = 0$$
$$x = -7 \quad \text{or} \quad x = 6$$

Domain will be for any x except -7 and 6

33, Find the exact value of $\log_7 \sqrt[3]{49}$

- Solution -

$$\log_7 \sqrt[3]{49} = \log_7 49^{1/3}$$

$$= \log_7 7^{2(\frac{1}{3})} = 2\frac{1}{3} \log_7 7$$

but $\log_7 7 = 1$

Therefore the answer is $2\frac{1}{3}$.

34, Rewrite the Logarithm $\log_3 142$ in terms of the common logarithm.

- Solution -

$$\log_3 142 = \frac{\log_{10} 142}{\log_{10} 3} = \frac{\log 142}{\log 3}$$

35, Use properties of logarithm to expand the expression as a sum, difference, and /or constant multiple of logarithms.

$$\log \frac{(x-1)^3}{y^2 z}$$

- Solution -

$\log \frac{(x-1)^3}{y^2 z}$. use the division rule 1^{st}.

$\log (x-1)^3 - \log y^2 z$

Use the product rule 2^{nd}.

$\log (x-1)^3 - (\log y^2 + \log z)$

Use the power rule.

$3 \log (x-1) - 2 \log y - \log z$

36, Condense the expression $7(\log x - \log y) + 2\log Z$.

-Solution-

$7(\log x - \log y) + 2\log Z$

$= 7\log x - 7\log y + 2\log Z$

$= \log x^7 - \log y^7 + \log Z^2$

$= \log \frac{x^7}{y^7} + \log Z^2$

$= \log \frac{x^7 Z^2}{y^7}$ answer.

37, Solve the system.

$$\begin{cases} x - y = -1 \\ x^2 - y = 5 \end{cases}$$

-Solution-

$x - y = -1 \Longrightarrow x = -1 + y$

Replace $x = -1 + y$ in $x^2 - y = 5$

$(-1+y)^2 - y = 5$

$y^2 - 2y + 1 - y = 5$

$y^2 - 3y + 1 = 5$. set the equation to 0.

$y^2 - 3y + 1 - 5 = 0$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4$ or $y = -1$.

Let's find x: $x = -1 + y$. When $y = 4$

$x = -1 + 4 = 3$. When $y = -1$; $x = -1 + -1$

$= -2$.

38, Solve the system:

$$\frac{7}{9}x + \frac{1}{9}y = \frac{8}{9}$$

$$7x + y = 8$$

- Solution -

$$\frac{7}{9}x + \frac{1}{9}y = \frac{8}{9}$$ Multiply by 9.

$$9 \cdot \frac{7}{9}x + 9 \cdot \frac{1}{9}y = 9 \cdot \frac{8}{9}$$

$$7x + y = 8$$

Since the 1^{st} equation is exactly the same as the second equation, the solution is "Dependent"

or $y = 8 - 7x$

39, Solve using any method.

$$2x + 9y = -9$$

$$9x - 8y = -19$$

- Solution -

Multiply the 1^{st} equation by -9 and the 2^{nd} equation by 2.

$$\begin{array}{r} -18x - 81y = 81 \\ 18x - 16y = -38 \\ \hline -97y = 43 \end{array}$$

Add Both Equations.

$$y = \frac{-43}{97}$$

Replace y with $\frac{-43}{97}$ in $2x + 9y = -9$

$$2x + 9\left(\frac{-43}{97}\right) = -9$$

$$2x - \frac{387}{97} = -9$$. Multiply by 97.

$$\begin{array}{r} 194x - 387 = -873 \\ +387 \quad\quad +387 \\ \hline \end{array}$$

$$194x = -486$$

$$x = \frac{-486}{194} = \frac{-243}{97}$$

40) Rewrite the exponential equation

$$5^{-3} = \frac{1}{125}$$ in Logarithmic form.

\- Solution -

$$\log_5 \tfrac{1}{125} = -3$$

41) Solve the equation:

$$2e^{x+3} = 5$$

\- Solution -

Isolate $e^{(x+3)}$ by dividing both sides by 2.

$$e^{(x+3)} = \frac{5}{2}$$

place $\ln$ in front of each term

$$\ln e^{(x+3)} = \ln \frac{5}{2}$$

By using properties of logs.

$(x+3)\ln e = \ln \frac{5}{2}$. But $\ln e = 1$.

$x+3 = \ln \frac{5}{2}$ and $x = -3 + \ln \frac{5}{2}$.

42) Solve the equation.

$$\log x + \log(x-3) = 1$$

\- Solution -

Make single log on the left side of the equation.

$\log_{10} x(x-3) = 1$. Base is 10.

using log Definition: $10^1 = x(x-3)$

$$10 = x^2 - 3x$$

$x^2 - 3x - 10 = 0$; $(x-5)(x+2) = 0$

$x = 5$ ✓ $\quad x = -2$ ✗

43, Solve for X in the equation given.

$$-2X = 4A - B$$

$$A = \begin{bmatrix} -2 & -3 \\ -1 & -8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -16 \\ 6 & -42 \end{bmatrix}$$

Solution.

Find $4A$ first by multiply each term in Matrix A by 4.

$$4A = \begin{bmatrix} -8 & -12 \\ -4 & -32 \end{bmatrix}.$$

find $-B$ by Multiplying Matrix B by -1.

$$-B = \begin{bmatrix} 2 & 16 \\ -6 & 42 \end{bmatrix}.$$

Now add $4A + {}^{-}B$. By adding the terms on the same location.

$$\begin{bmatrix} -8+2 & -12+16 \\ -4+{}^{-}6 & -32+42 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ -10 & 10 \end{bmatrix}.$$

But $-2X = \begin{bmatrix} -6 & 4 \\ -10 & 10 \end{bmatrix}$. Therefore

By dividing each term by -2, you'll get X.

$$X = \begin{bmatrix} 6 & -2 \\ 5 & -5 \end{bmatrix}$$

44, Find the determinant of the matrix

$$\begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -5 & -\frac{1}{3} \end{bmatrix}$$

_ Solution _

$$\text{Det} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

$$\text{Det} \begin{bmatrix} 5/3 & -4/3 \\ -5 & -1/3 \end{bmatrix} = \left(\frac{5}{3} \cdot \frac{-1}{3}\right) - \left(-5 \cdot \frac{-4}{3}\right)$$

$$= \frac{-5}{9} - \frac{20}{3}$$

$$= \frac{-5}{9} - \frac{60}{9} = \frac{-65}{9}.$$

45, If possible find AB.

$$A = \begin{bmatrix} 8 & -4 \\ -6 & 1 \\ 6 & 2 \end{bmatrix}; \quad B = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

_ solution _

$$\begin{bmatrix} 8 & -4 \\ -6 & 1 \\ 6 & 2 \end{bmatrix} \times \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \times {}^-4 + {}^-4 \times {}^-3 \\ -6 \times {}^-4 + 1 \times {}^-3 \\ 6 \times {}^-4 + 2 \times {}^-3 \end{bmatrix} = \begin{bmatrix} -32 + 12 \\ 24 - 3 \\ -24 + {}^-6 \end{bmatrix}$$

$$= \begin{bmatrix} -20 \\ 21 \\ -30 \end{bmatrix}$$