

Quick Review on AP Physics 1 or College Physics Detailed Solved Examples

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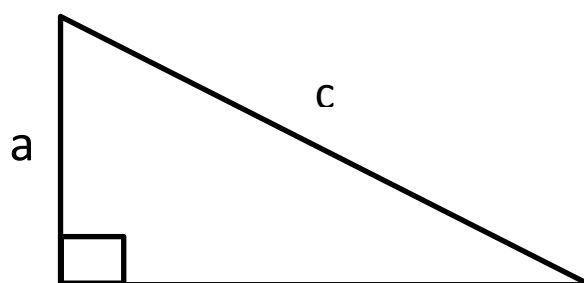
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Math Background

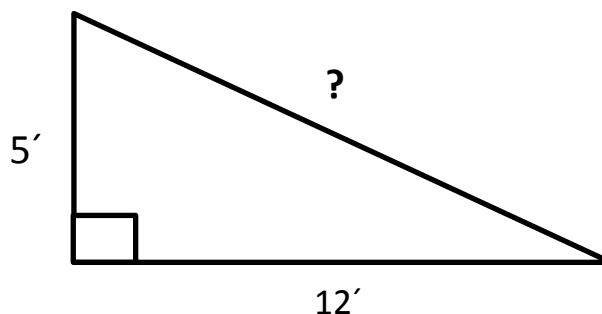
– For physics –

Pythagorean theorem: It can only be applied for right angle triangles.



$$a^2 + b^2 = c^2$$

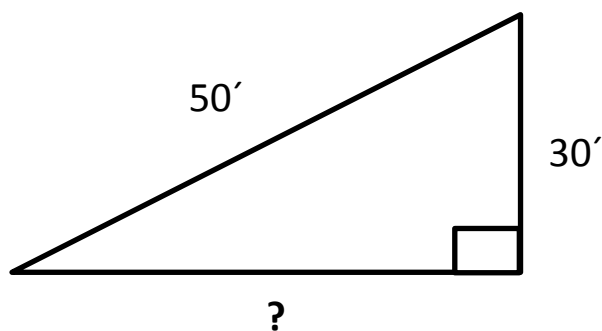
Example 1:



–Solution–

$$\text{Hypo tenus} = \sqrt{(5^2 + 12^2)} = \sqrt{25 + 144} = \sqrt{169} = 13$$

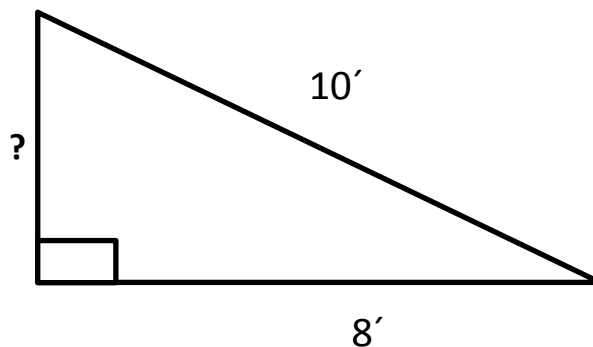
Example 2:



To find any of the smaller side of a right triangle:

$$= \sqrt{50^2 - 30^2} = \sqrt{2500 - 900} = \sqrt{1600} = 40'$$

Example 3:



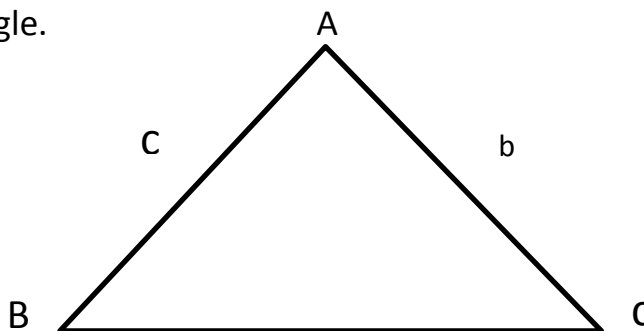
-Solution-

Since we're looking for the small side:

$$=\sqrt{10^2 - 8^2}=\sqrt{100 - 64}=\sqrt{36}=6$$

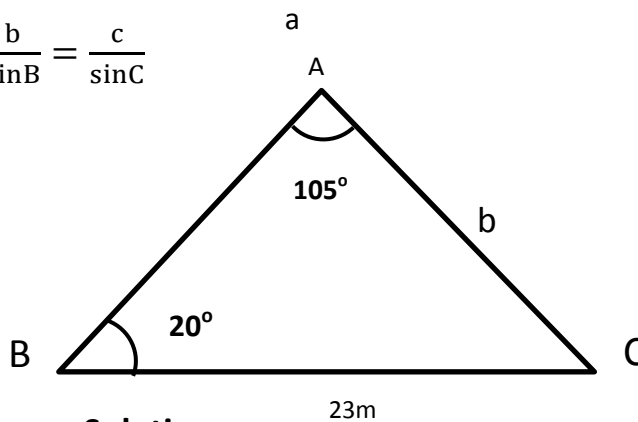
~law of sine~

It works for any type of triangle.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1:



-Solution-

$$\frac{23}{\sin 105^\circ} = \frac{b}{\sin 20^\circ} \quad \sin 105^\circ = 0.94, \sin 20^\circ = 0.34$$

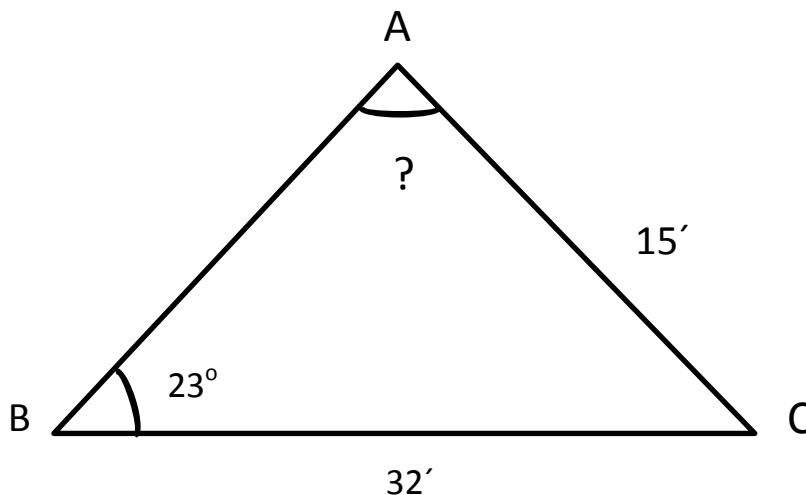
$$\frac{23}{0.94} = \frac{b}{0.34}$$

Cross multiply: $0.94b=7.82$

Divide both sides by 0.94

$$b=8.06\text{m}$$

Example 2:



-Solution-

$$\frac{32}{\sin A} = \frac{15}{\sin 23^\circ}$$

$$; \sin 23 = 0.39$$

$$\frac{32}{\sin A} = \frac{15}{0.39}$$

Cross multiply:

$$12.48 = 15 \sin A$$

Divide both sides by 15.

$$\sin A = 0.832$$

To find the measure of angle A

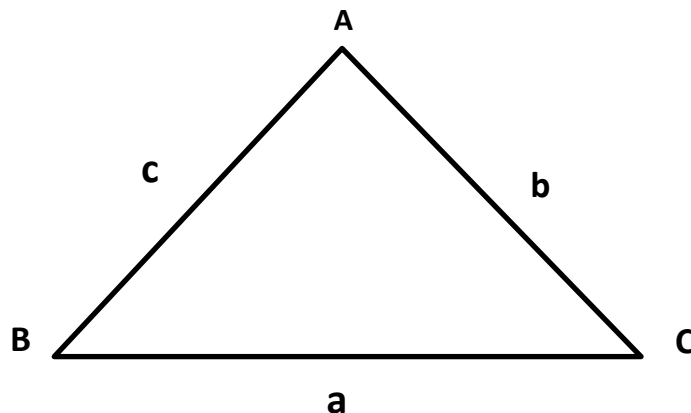
Press shift or 2nd function

Sin 0.832= (on the calculator)

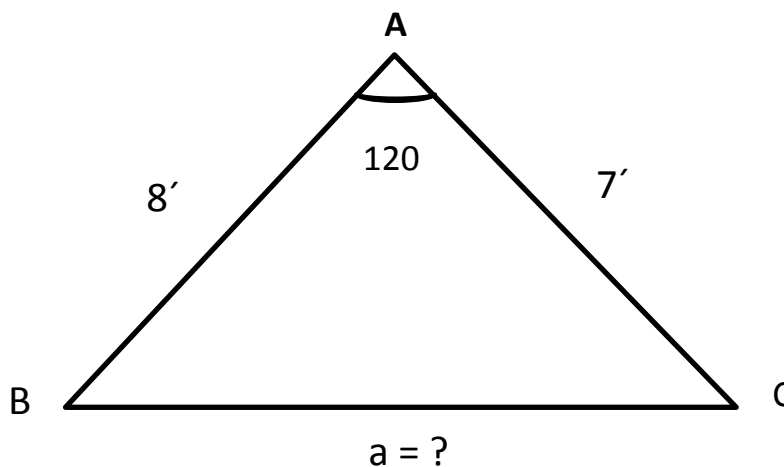
$$M < A = 56.30^\circ$$

– Law of cosine–

It works for any type of triangle. it is mostly used when given 2 sides and an angle between them, and you are asked to find the 3rd side.



Example:



–Solution–

$$a = \sqrt{(8^2 + 7^2 - 2 \times 8 \times 7 \cos 102^\circ)}$$

$$= \sqrt{64 + 49 - 112 \cos 102^\circ}$$

$$; \text{ But } \cos 102^\circ = -0.208$$

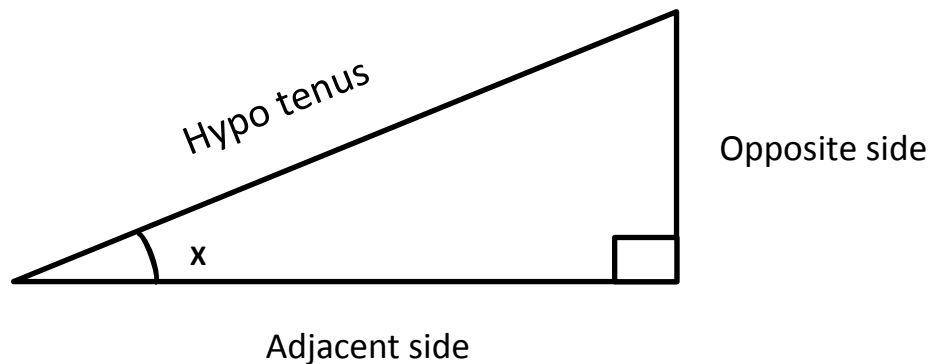
$$a = \sqrt{64 + 49 - 112(-0.208)}$$

$$= \sqrt{113 + 112 \times 0.208}$$

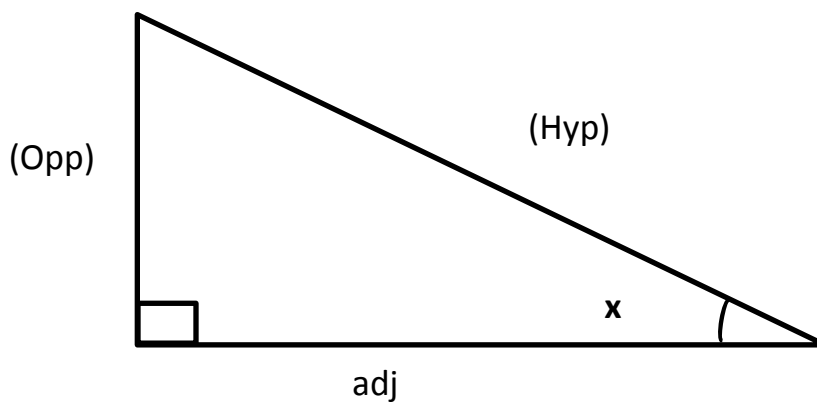
$$= \sqrt{136.24} = a = 11.67$$

-Trig Ratios-

Trigonometric ratios work for right angle triangle only.



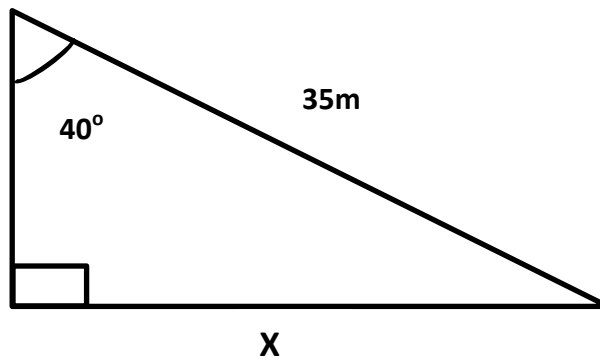
The side opposite to the angles is the opposite side. The hypotenuse is the longest side. Therefore the remaining side is the adjacent.



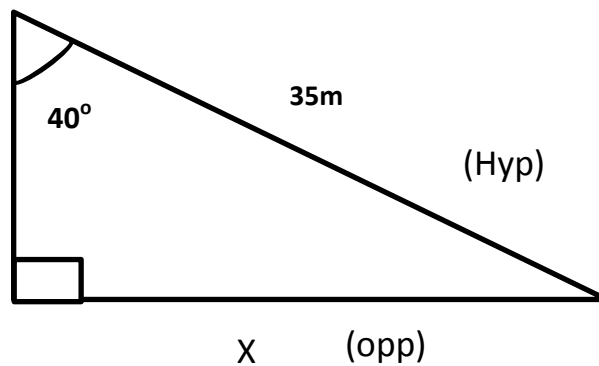
$$\sin x = \frac{\text{Opp}}{\text{Hyp}}$$

$$\cos x = \frac{\text{adj}}{\text{Hyp}}$$

$$\tan x = \frac{\text{Opp}}{\text{adj}}$$

Example 1:**-Solution-**

Label the sides that are part of the problem only.



The trig ratio that has to do with opposite and hypotenus is the SIN.

$$\sin 40^\circ = \frac{x}{35}$$

$$\sin 40^\circ = 0.64$$

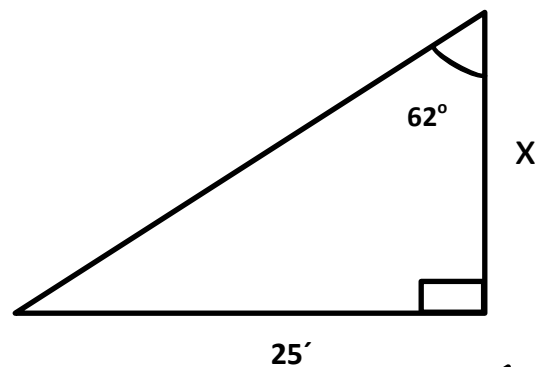
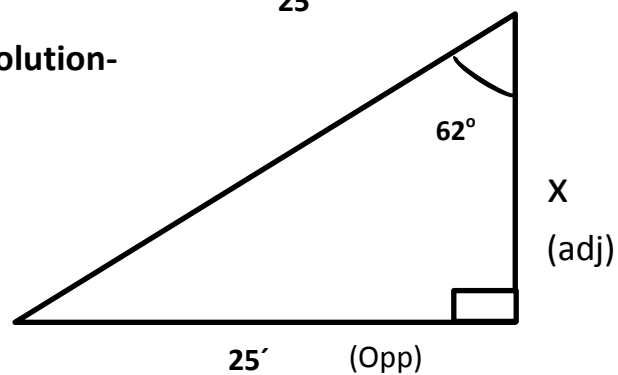
$$\frac{0.64}{1} = \frac{x}{35}$$

cross multiply.

$$X = 22.4\text{m}$$

Example 2:

Label the sides that are part of the problem.

**-Solution-**

The trig ratio that has to do with the opposite and adjacent is the tangent.

$$\tan 60^\circ = \frac{25}{x}$$

$$\tan 60^\circ = 1.88$$

$$\frac{1.88}{1} = \frac{25}{x}$$

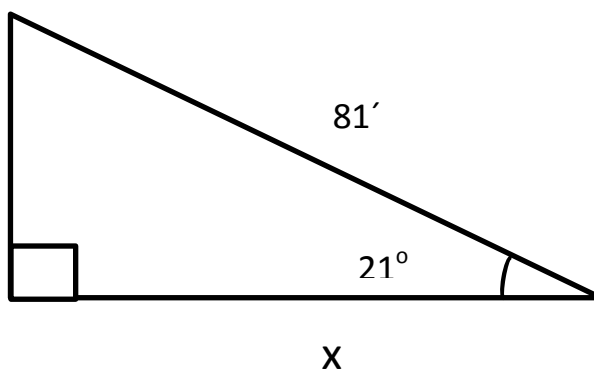
cross multiply:

$$1.88x = 25$$

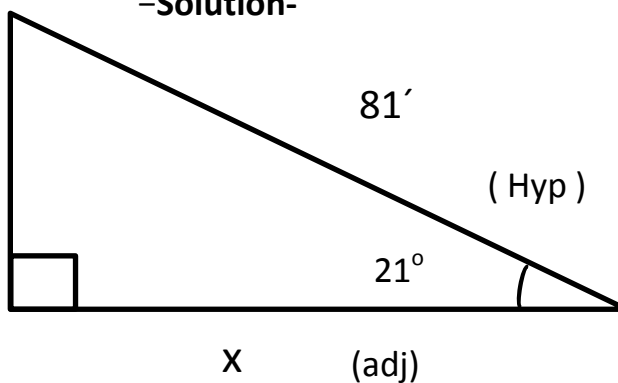
Divide both sides by 1.88

$$X = 13.29'$$

Example 3:



-Solution-

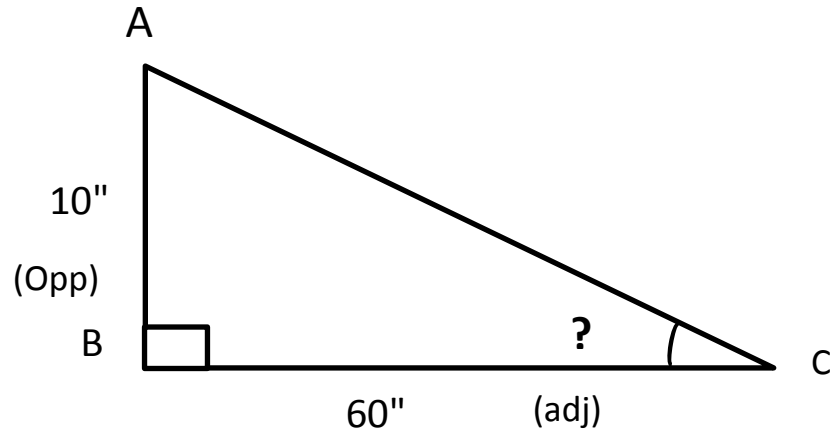


$$\cos 21^\circ = \frac{x}{81}$$

$$\cos 21^\circ = 0.933$$

$$\frac{0.933}{1} = \frac{x}{81}$$

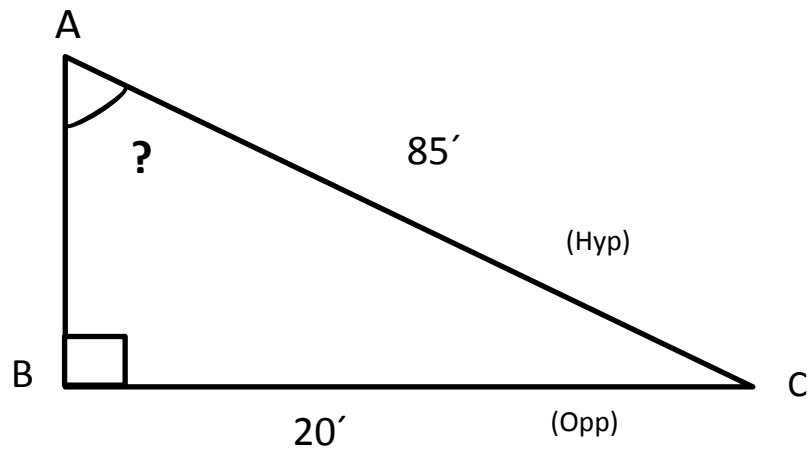
$$X = 75.57'$$

Example 4:**-Solution-**

$$\tan c = \frac{10}{60} = 0.166$$

To find $m \angle c$, press 2nd function or shift tan 0.166

$$= m \angle c = 9.42^\circ$$

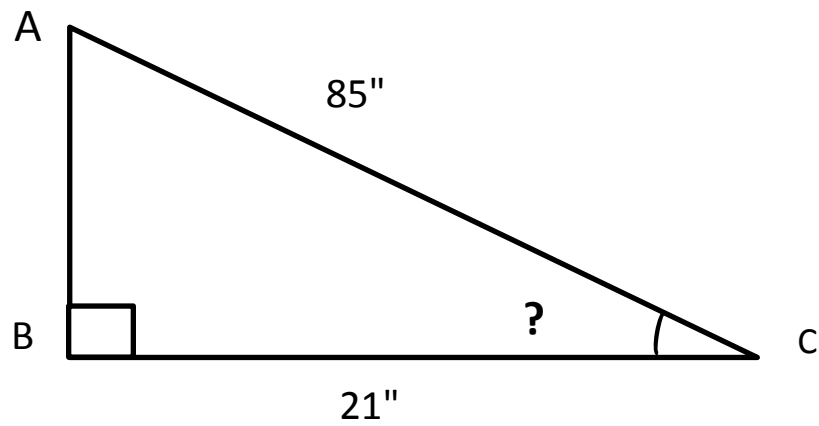
Example 5:**-Solution-**

$$\sin A = \frac{\text{opp}}{\text{Hyp}}$$

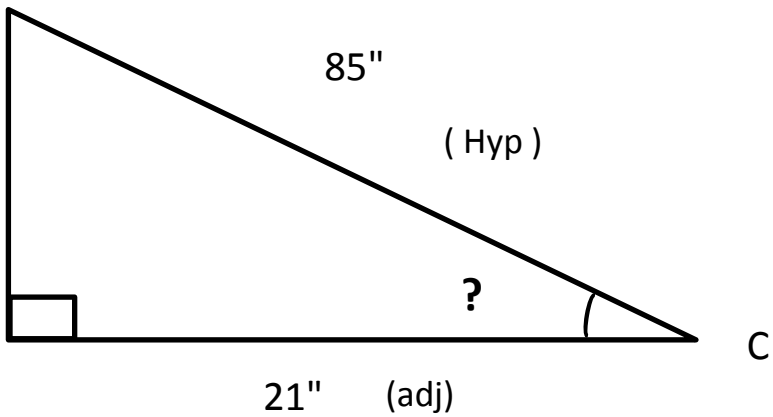
$$\sin A = \frac{20}{85} = 0.23$$

$$m\angle C = 2^{\text{nd}} \sin 0.23 \\ = 13.29^\circ$$

Example 6:



-Solution-



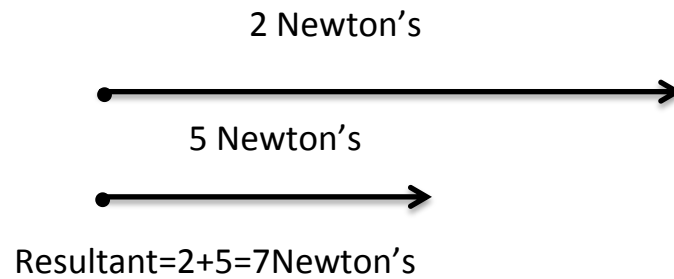
$$\cos C = \frac{21}{85} = 0.24 \quad \therefore m\angle C = 76^\circ$$

-Vectors-

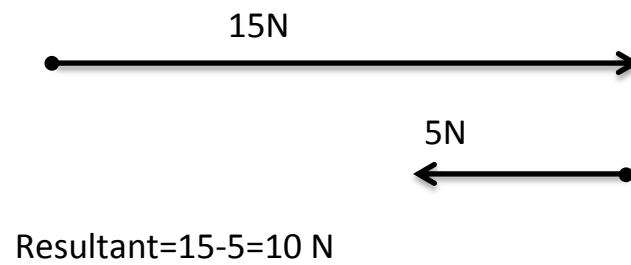
Vectors are quantities that have directions & magnitudes. Examples are forces, velocities, accelerations and so on.

Resultant of vectors:

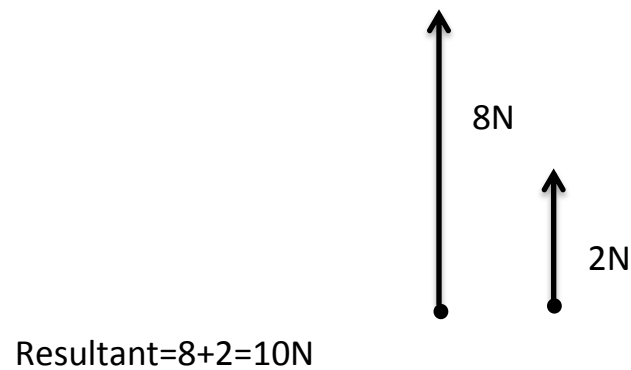
Example 1:

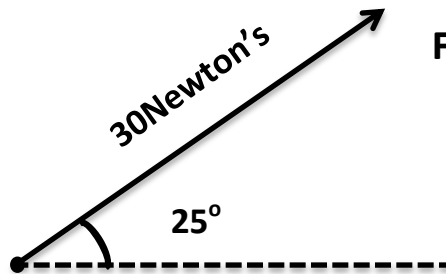


Example 2:



Example 3:



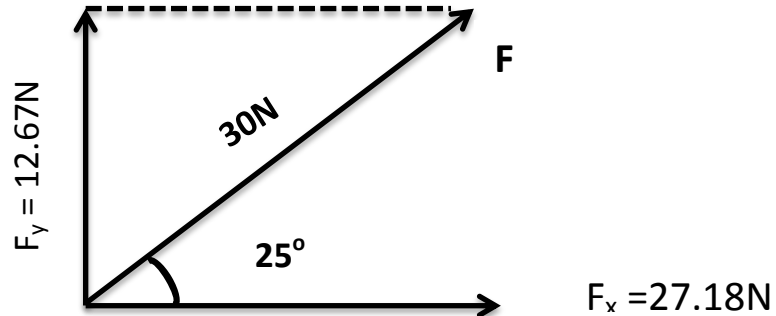
Example 4:

Find the X and Y components of F.

-Solution-

$$F_x = 30 \cos 25^\circ = 27.18\text{N}$$

$$F_y = 30 \sin 25^\circ = 12.67\text{N}$$

**Example 5:** Find the net displacement.

-Solution-

Break down each displacement into X and Y components.

$$d_1(x) = 50 \cos 22^\circ = 46.35\text{m}; \quad d_1(y) = 50 \sin 22^\circ = 18.73\text{m}$$

$$d_1 = (46.35, 18.73)$$

d_2 has only an X component.

$$(85, 0)$$

d_3 the angle is below the horizontal,

therefore it is negative.

$$d_3(X) = 30 \cos(-27^\circ) = 26.73\text{m}$$

$$d_3(Y) = 30 \sin(-27^\circ) = -13.61$$

$$d_3(26.73, -13.61)$$

Now add all the X-components together and all the Y-components together.

X components: $46.35 + 85 + 26.73$

$$= 158.08\text{m}$$

Y components: $18.73 + (-13.16)$

$$= 5.12\text{m}$$

Net displacement = $(158.08, -5.12)$

To find its magnitude = $\sqrt{(158.08^2 + 5.12^2)}$

$$= 158.16\text{m}$$

To find its direction: $2^{\text{nd}} \tan \frac{5.12}{158.08}$

$$= 1.80^\circ$$

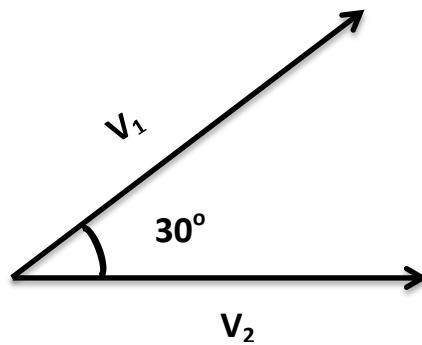
Net displacement is AB

Resultant of vectors

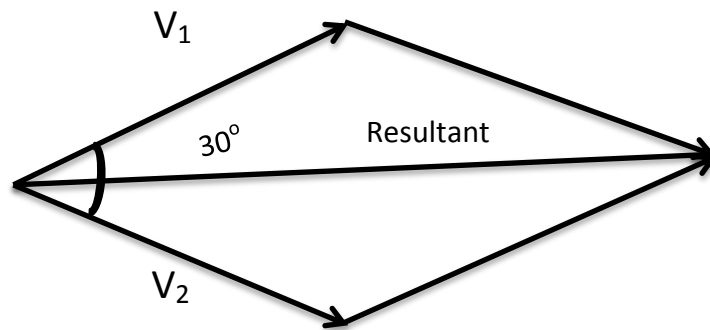
Starting from the

Same point

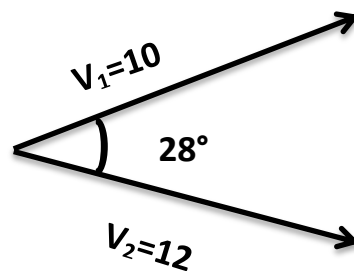
Example 1:



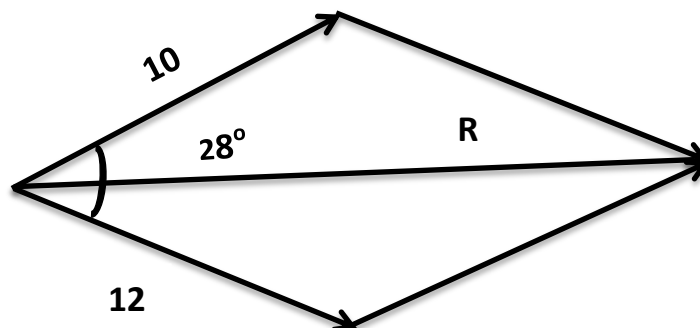
Resultant is the diagonal of the parallelogram.



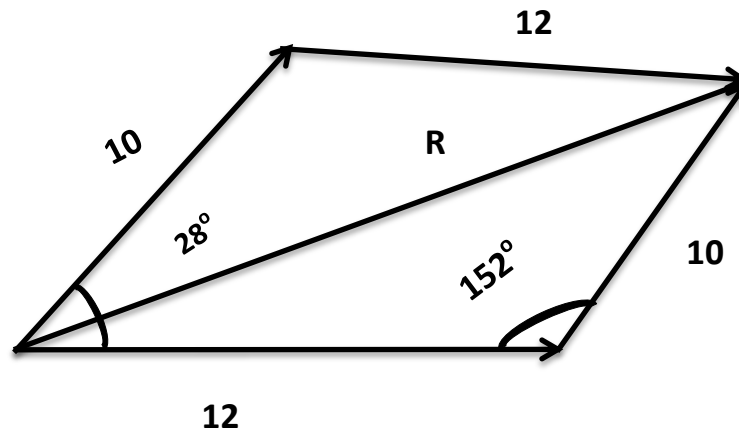
Example 2:



Find the Resultant vector:



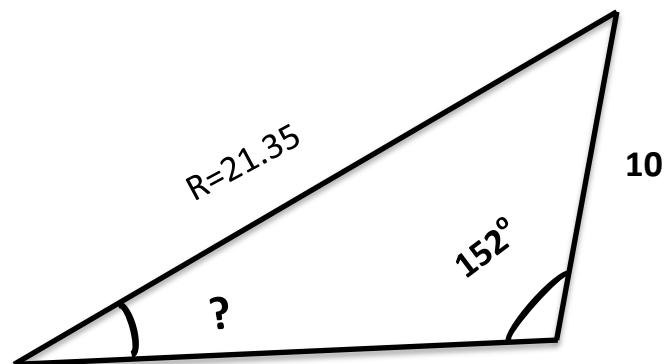
Opposite sides of a parallelogram are congruent or equal and opposite angles are equal and adjacent angles add up to 180°



Using the law of cosine:

$$\begin{aligned}
 R &= \sqrt{(10^2 + 12^2 - 2 \times 10 \times 12 \cos 152^\circ)} \\
 &= \sqrt{(100 + 144 - 240 \cos 152^\circ)} \\
 &= \sqrt{244 + 211.90} \\
 &= \sqrt{455.90} = 21.35 \\
 R &= 21.35
 \end{aligned}$$

To find the direction of R which means the angle R makes with the horizontal:



Use law of sin: $\frac{21.35}{\sin 152^\circ} = \frac{10}{\sin x}$

$$\frac{21.35}{0.469} = \frac{10}{\sin x}$$

Cross multiply:

$$21.35 \sin X = 4.69$$

$$\sin X = \frac{4.69}{21.35}$$

$$= 0.21$$

$$M < X = 2^{\text{nd}} \sin 0.21 = 12.12^\circ$$

Average velocity

$$V_a = (V_i + v_f) / 2 \quad (1)$$

Where

V_a = average velocity (m/s)

V_i = initial velocity (m/s)

V_f = final velocity (m/s)

Final Velocity

$$V_f = V_i + at \quad (2)$$

Where

a = acceleration (m/s^2)

t = time taken (s)

$$V_f^2 - V_i^2 = 2as$$

Distance Traveled

$$S = (v_i + v_f) t / 2 \quad (3)$$

Where

S = distance traveled (m)

Alternative:

$$S = V_i t + \frac{1}{2} at^2 \quad (3b)$$

Acceleration

$$a = (v_f - v_i) / t \quad (4)$$

Kinematics(speed, velocity, displacement,and acceleration sample problems)

1. An airplane accelerates down a runway at 3.20 m/sec^2 for 32.8 sec until this finally lifts off the ground. Determine the distance traveled before takeoff.
2. A car start form rest and accelerates uniformly over a time of 5.21Second for a distance of 110 determines the acceleration of the car.
3. Upton chuck is riding the giant drop at great America. If Upton free falls for 2.60 second. What will be this final velocity and how far will he fall.
4. A race of the car accelerates uniformly from 18.5 m/s to 46.1 m/s in 2.47 determine the acceleration of the and distance traveled.
5. A feather is dropped on the moon from a height of 1.40 meters. The acceleration of gravity of the moon is 1.67 m/s^2 . Determine the time for feather to fall to the surface of the moon.
6. Rocket- powered sleds are used to test the human response to accelerated. If a rocket-powered sled is accelerate to a speed of 444 m/s in 1.83 second, then what is the acceleration and what is the distance that the sled travels?
7. A bike accelerates uniformly from rest to a speed of 7.10 m/s over a distance of 35.4 m. determines the acceleration of the bike?
8. An engineer is designing the runway for an airport. One of the plane that will use the airport, the lowest acceleration rate is likely to be 3 m/s^2 .The takeoff speed for plane will be 65 m/s . assuming this minimum acceleration. What is the minimum allowed length for the runway?
9. A car traveling at 22.4 m/s skids to a stop in 2.55 determine the skidding distance of the car (assume uniform acceleration)
10. A kangaroo is capable of jumping to a height of 2.62 determine the takeoff speed of the kangaroo.
11. If Michael Jordan has a vertical leap of 1.29m, than what are the takeoff speed and his hang time (total time to move upwards to the peak and then return to the ground)?
12. A bullet leaves a rifle with a muzzle velocity of 521 m/ while accelerating through the barrel of the rifle, the bullet moves a distance of 0.840m. Determines the acceleration of the bullet.(Assume uniform acceleration)
13. A baseball is popped straight up into the air and has a hang- time of 6.25 second. Determine the height to which the ball rises before it reaches its peak.(hint: the time to rise to the peak is half the total hang time)
14. The observation deck of tall skyscraper 370 m above the street. Determines the time required for a penny to free – fall from the deck to the street below.
15. A bullet is moving at a speed of 367 m/s when it embeds into a lump of moist clay. The bullet penetrates for a distance of 0.0621 determines acceleration of the bullet while moving into the clay. (Assume uniform acceleration).

-Solution to kinematics-

1. Given: $a=+3.2\text{m/s}$

$$t=32.8\text{s}$$

$$V_0 \text{ or } V_i = 0$$

$$S=?$$

The formula that involves what's given is:

$$S = V_i t + \frac{1}{2} a t^2$$

$$= 0 \times 32.8 + 0.5 \times 3.2 \times 32.8^2$$

$$= 1720\text{m}$$

2. Given:

$$S=110\text{m}, \quad t=5.21\text{sec}$$

$$V_i = 0 \text{ m/s}$$

$$a = ?$$

The formula that involves what's given is:

$$S = V_i t + \frac{1}{2} a t^2$$

$$110 = 0 \times 5.21 + 0.5 \times a \times 5.21^2$$

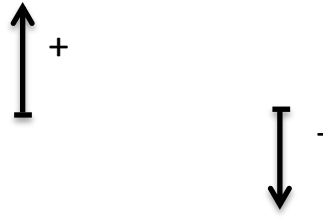
$$110 = 0.5 \times a \times 5.21^2$$

$$110 = 13.57a$$

$$a = 110/13.57$$

$$= 8.1\text{m/s}$$

3. Given:



Since the direction is downward,

$$a = g = -9.8\text{m/s}^2$$

$$t = 2.6\text{sec}$$

$$V_i = 0\text{m/s}$$

$$S = ?$$

$$V_f = ?$$

$$S = V_i t + \frac{1}{2} a t^2$$

$$= 0 \times 2.6 + 0.5 \times -9.8 \times 2.6^2$$

$$= -33.1\text{m}$$

direction is downward.

To find V_f , use the formula that involves V_f .

$$a = \frac{V_f - V_i}{t}$$

$$-9.8 = \frac{V_f - 0}{2.6} \quad \text{Cross multiply}$$

$$V_f = -9.8 \times 2.6$$

$$= -25.5\text{m/s}$$

4. Given:

$$V_i = 18.5 \text{ m/s}$$

$$V_f = 46.1 \text{ m/s}$$

$$t = 2.47 \text{ sec}$$

$$S = ?$$

$$a = ?$$

It is easier to find "a" first, the formula that involves a, V_f , V_i is:

$$a = \frac{V_f - V_i}{t}$$

$$a = \frac{46.1 - 18.5}{2.47}$$
$$= 11.21 \text{ m/s}^2$$

To find s,

$$S = V_i t + \frac{1}{2} a t^2$$
$$= 18.5 \times 2.47 + 0.5 \times 11.21 \times 2.47^2$$
$$= 79.84 \text{ m}$$

5. Given: $V_i = 0\text{m/s}$

$$S = -1.40\text{m}$$

$$a = -1.67\text{m/s}^2$$

$$t = ?$$

$$S = V_i t + \frac{1}{2} a t^2 \text{ or } V_o t + \frac{1}{2} a t^2$$

$$-1.40 = 0 \times t + 0.5 \times -1.67 t^2$$

$$-1.40 = -0.835 t^2 \quad ; \text{Divide both side by } -0.835$$

$$1.67 = t^2$$

$$t = \sqrt{1.67} = 1.29\text{sec}$$

6. Given: V_i or $V_o = 0\text{m/s}$

$$V_f = 444 \text{ m/s}$$

$$t = 1.83 \text{ s}$$

$$a = ?$$

$$s = ?$$

$$a = \frac{V_f - V_i}{t}$$

$$a = \frac{444 - 0}{1.83}$$

$$= 242.62 \text{ m/s}^2$$

$$S = V_i t + \frac{1}{2} a t^2$$

$$= 0 \times 1.83 + 0.5 \times 242.62 \times 1.83^2$$

$$= 406.25\text{m}$$

7. Given:

$$V_i = 0 \text{ m/s}$$

$$V_f = 7.10 \text{ m/s}$$

$$s = 35.4 \text{ m}$$

$$a = ?$$

The formula that involves V_i , V_f , a and s is:

$$V_f^2 - V_i^2 = 2as$$

$$7.10^2 - 0^2 = 2a \times 35.4$$

$$50.41 = 70.8a$$

$$a = 0.71 \text{ m/s}^2$$

8. Given:

$$V_i = 0 \text{ m/s}$$

$$V_f = 65 \text{ m/s}$$

$$a = 3 \text{ m/s}^2$$

$$S = ?$$

$$V_f^2 - V_i^2 = 2as$$

$$65^2 - 0 = 2 \times 3 \times S$$

$$4225 = 6s$$

$$S = 704.16 \text{ m}$$

9 Given:

$$V_i = 22.4 \text{ m/s}$$

$$V_f = 0 \text{ m/s}$$

$$t = 2.55 \text{ s}$$

$$S = ?$$

The formula that involves V_i , V_f , t and S is:

$$\begin{aligned} S &= \frac{(V_i + V_f) t}{2} \\ &= \frac{(0 + 22.4)}{2} \times 2.55 \\ &= 11.2 \times 2.55 = 28.56 \text{ m} \end{aligned}$$

10 Given:

$$a = -9.8 \text{ /s}^2$$

$$V_f = 0 \text{ m/s}$$

$$S = 2.62 \text{ m}$$

$$V_i = ?$$

$$V_f^2 - V_i^2 = 2aS$$

$$0^2 - V_i^2 = 2 \times -9.8 \times 2.62$$

$$-V_i^2 = -51.35$$

$$V_i^2 = 51.35$$

$$V_i = \sqrt{51.35}$$

$$= 7.16 \text{ m/s}^2$$

11. Given:

$$a = -9.8 \text{ m/s}^2$$

$$V_f = 0 \text{ m/s}$$

$$S = 1.29 \text{ m}$$

$$V_i = ?$$

$$t = ?$$

$$V_f^2 - V_i^2 = 2as$$

$$0^2 - V_i^2 = 2 \times -9.8 \times 1.29$$

$$-V_i^2 = -25.28$$

$$V_i^2 = 25.28$$

$$V_i = \sqrt{25.28} = 5.02 \text{ m/s}^2$$

Time for the peak

$$a = \frac{V_f - V_i}{t}$$

$$-9.8 = \frac{0 - 5.02}{t} \quad \text{Cross multiply:}$$

$$-9.8t = -5.02$$

$$t = 0.512 \text{ sec}$$

Hang time = 2 x peak time

$$= 2 \times 0.512$$

$$= 1.02 \text{ sec}$$

12. Given:

$$V_i = 0 \text{ m/s}$$

$$V_f = 521 \text{ m/s}$$

$$S = 0.840 \text{ m}$$

$$a = ?$$

$$V_f^2 - V_i^2 = 2as$$

$$521^2 - 0^2 = 2a \times 0.840$$

$$271441 = 1.68a$$

$$a = 161572.02 \text{ m/s}^2$$

13. Given:

$$a = -9.8 \text{ m/s}^2$$

$$V_f = 0 \text{ m/s}$$

$$t = 3.13 \text{ sec}$$

$$S = ?$$

$$a = \frac{V_f - V_i}{t}$$

$$-9.8 = \frac{0 - V_i}{3.13}$$

$$-30.67 = -V_i$$

$$V_i = 30.67 \text{ m/s}$$

Now, use $V_f^2 - V_i^2 = 2aS$

$$0 - 30.67^2 = 2 \times -9.8 \times S$$

$$S = 48 \text{ m}$$

14 Given:

$$V_i = 0 \text{ m/s}$$

$$S = -370 \text{ m}$$

$$a = g = -9.8 \text{ m/s}^2$$

$$t = ?$$

$$S = V_i t + \frac{1}{2} a t^2$$

$$-370 = 0 \times t + 0.5 \times -9.8 t^2$$

$$t^2 = 75.51$$

$$t = 8.68 \text{ sec}$$

15 Given:

$$V_i = 367 \text{ m/s}$$

$$V_f = 0 \text{ m/s}$$

$$S = 0.0621 \text{ m}$$

$$a = ?$$

$$V_f^2 - V_i^2 = 2as$$

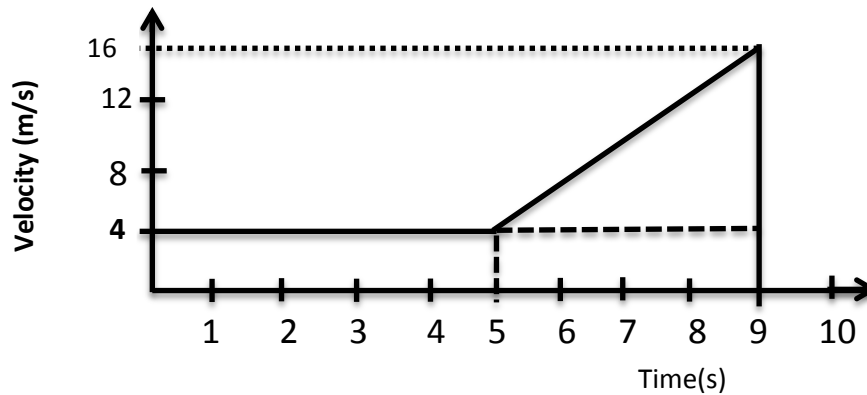
$$0^2 - 367^2 = 2a \times 0.0621$$

$$-134689 = 0.124a$$

$$a = -1086201.6 \text{ m/s}^2$$

- displacement and velocity –

Example1:



Find the total displacement.

Displacement = total area of velocity versus time.

= area of the rectangle + area of the triangle

Area of the rectangle = $l \times w$

$$= 9 \times 4$$

$$= 36 \text{ m.}$$

Area of the triangle = $\frac{b \times h}{2}$

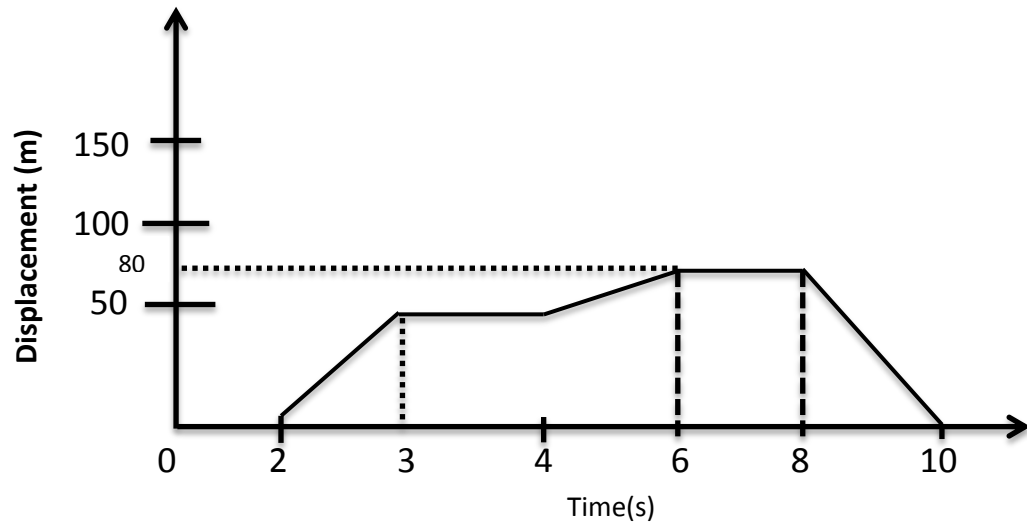
$$= \frac{4 \times 12}{2}$$

$$= 24 \text{ m.}$$

Total displacement = $36 \text{ m} + 24 \text{ m}$

$$= 60 \text{ m.}$$

Example 2:



a, Find the velocity from 2 to 3 seconds .

$$v = \text{slope} = \frac{50-0}{3-2} = \frac{50}{1} = 50 \text{ m/s}^2$$

b, Find the velocity from 3 to 4 seconds.

$$v = \text{slope} = 0, \text{ since it is a horizontal line.}$$

c, Find the velocity from 8 to 10 seconds.

$$v = \text{slope} = \frac{0-80}{10-8} = -\frac{80}{2} = -40 \text{ m/s}^2$$

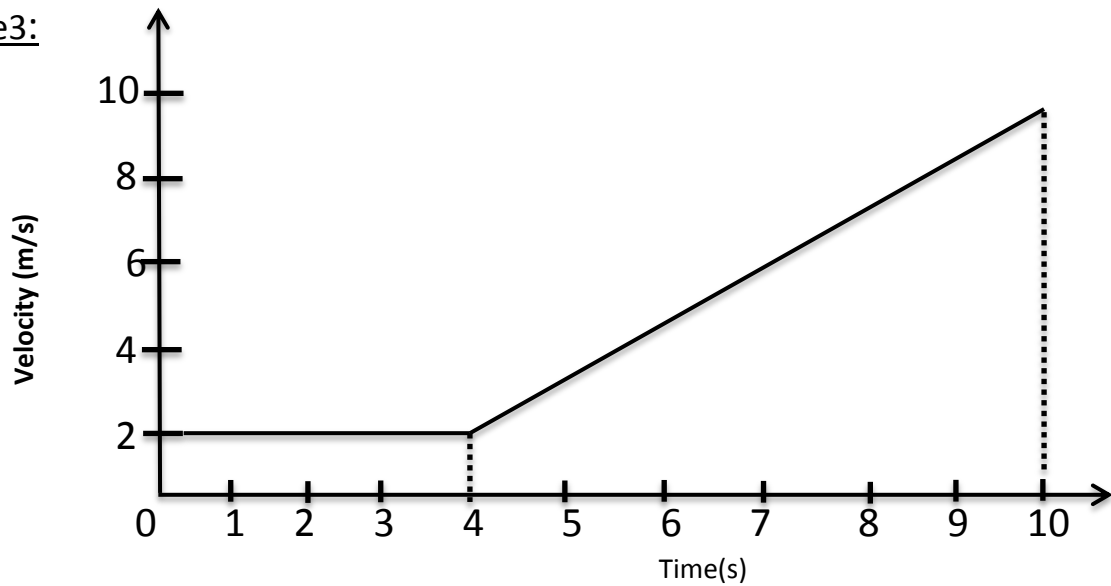
d, what is the mean velocity?

$$\text{mean velocity} = \frac{\text{total displacement}}{\text{total time}}$$

$$= \frac{50+0+30+80}{8}$$

$$= \frac{160}{8} = 20 \text{ m/s}^2$$

Example3:



a, find the acceleration from 0 to 4 second.

$a = \text{slope of the velocity}$

$= 0$ because it is a horizontal line.

b, find the acceleration from 4 to 10 second.

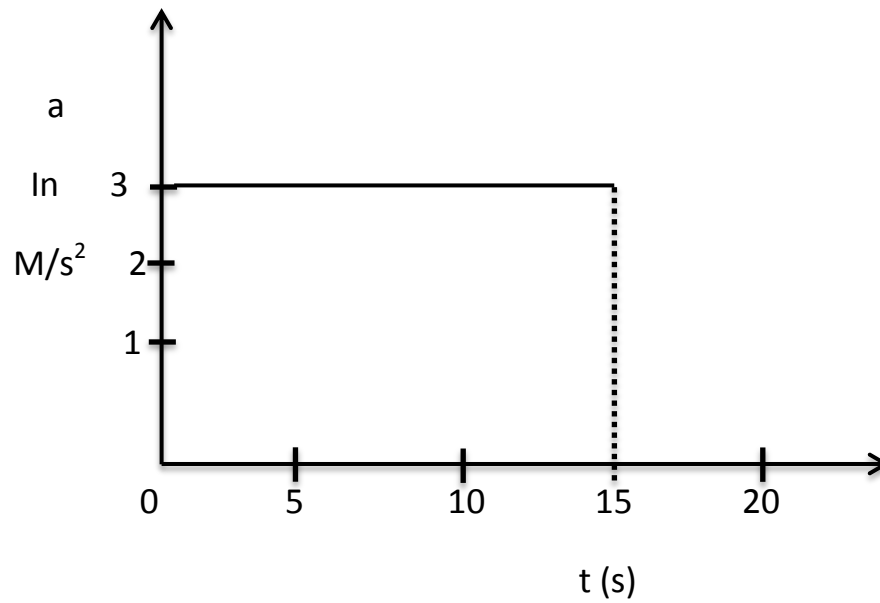
$$a = \text{slope} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$(4, 2), (10, 10)$

$$= \frac{10 - 2}{10 - 4}$$

$$= \frac{8}{6}$$

$$= 1.33 \text{ m/s}^2$$

Example 4:

Find the change in velocity

Velocity = area of the graph of acceleration

= area of the rectangle

= $15 \text{ second} \times 3 \text{ m/s}^2$

= 45 m/s^2

–Projectile Motion–

Object thrown at

~An angle~

$a_x=0 \implies$ Horizontal acceleration is 0

\implies Horizontal velocity does not change

$a_y = -g \implies$ the vertical acceleration is due to gravity -9.8m/s^2 . It is negative because, it is down ward.

$$v_x = v_{0x} + a_{xt} \implies \sin a \quad a_x = 0$$

$\therefore v_x = v_{0x} \implies$ Horizontal velocity = initial velocity

$V_y = v_{0y} - gt \implies$ vertical velocity = initial vertical velocity $-9.8 \times$ time.

$$V_y^2 = v_{0y}^2 - 2g \Delta y$$

“ Δy the change in height .

$y - y_0$ where y_0 is the Initial height.”

$$Y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

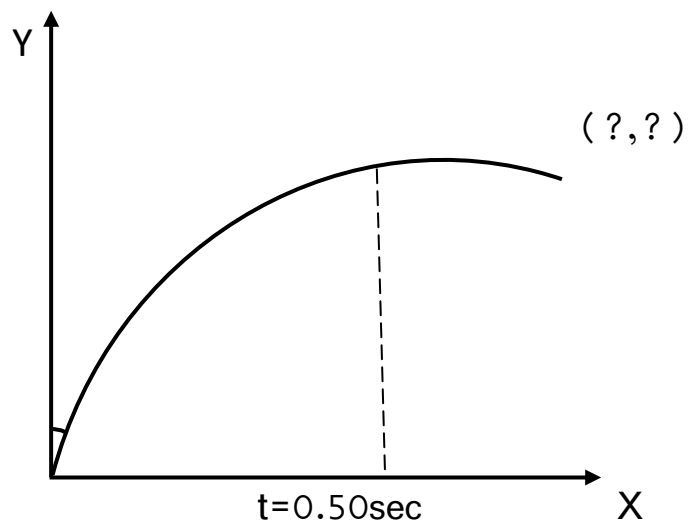
$$v_{0x} = v_0 \cos \theta$$

$$V_{0y} = v_0 \sin \theta$$

$X = x_0 + v_{0x} t \implies x_0$ is the Initial Horizontal displacement.

Example 1: A projectile is launched from the Origin with an initial speed of 20 m/s at an angle of 35° above the horizontal. Find the x and y position of the projectile at $t = 0.50$ sec.

–Solution–



$$V_{0x} = V_0 \cos \theta = 20 \cos 35^\circ = 16.38 \text{ m/s}$$

$$V_{0y} = V_0 \sin \theta = 20 \sin 35^\circ = 11.47 \text{ m/s}$$

Since we're looking for x and y.

The formulas are:

$$X = x_0 + v_{0x} t$$

$$X = 0 + 16.38 \times 0.50 = 8.19 \text{ m}$$

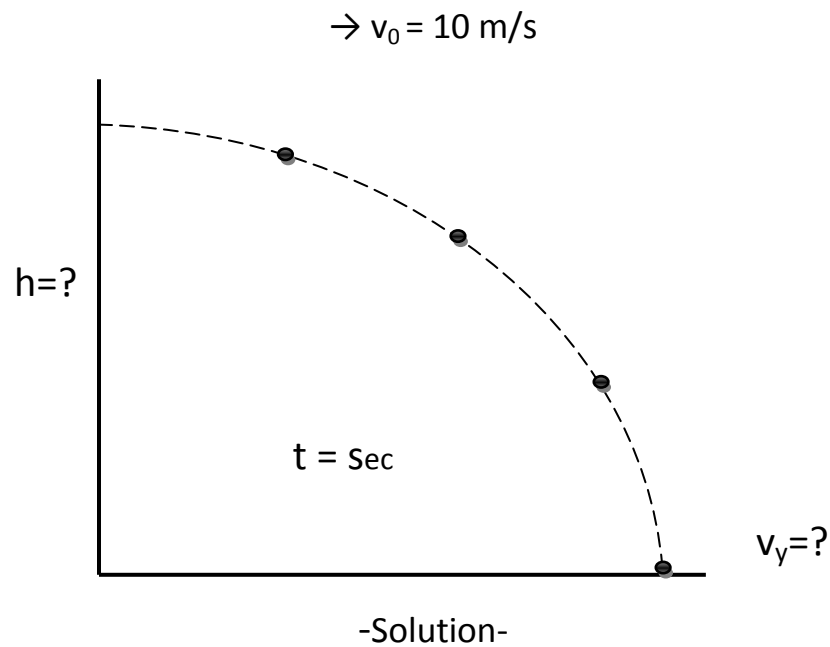
$$Y = y_0 + V_{0y}t - \frac{1}{2} g t^2$$

$$Y = 0 + 11.47 \times 0.50 - 0.5 \times 9.8 \times 0.50^2$$

$$Y = 4.51 \text{ m}$$

$$(8.19 \text{ m}, 4.51 \text{ m})$$

Example 2 : Alia throws the ball to the + x direction with an initial velocity of 10 m/s . Calculate the height that object is thrown and v_y component of the velocity after it hits the ground. If the time elapsed during the motion is 5 seconds.



In the vertical direction, we have the following equations:

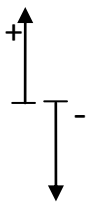
$$h = \frac{1}{2} g t^2 + v_{0y} t + h_0$$

$$v_{0y} = 0, \quad h_0 = 0$$

“g” is negative

$$h = \frac{1}{2} g t^2$$

$$= 0.5 \times -9.8 \times 5^2 = -125 \text{ m}$$

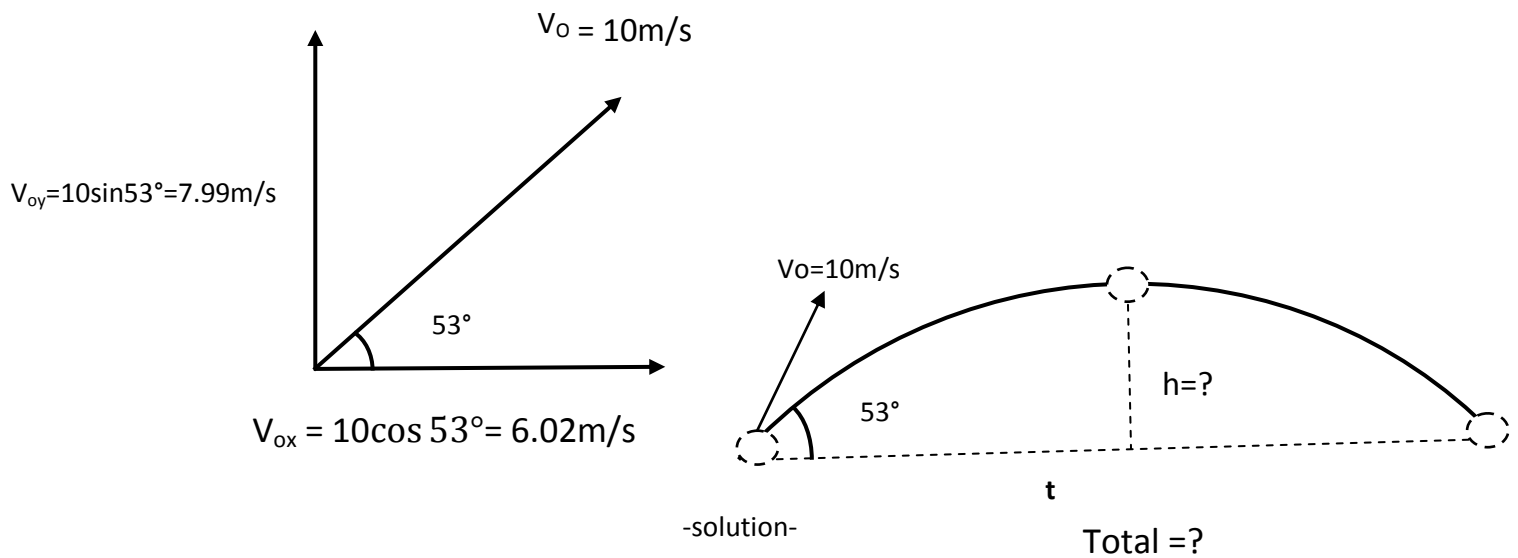


$$V_y = V_{0y} - g t$$

$$V_y = 0 - 9.8 \text{ m/s}$$

$$V_y = -49 \text{ m/s}$$

Example 3: John kicks the ball in a projectile motion at an angle of 53° to horizontal its initial velocity is 10 m/s. find the maximum height it reaches, horizontal displacement and total time for this motion.



Vertical motion formulas:

$$V_y = v_{0y} - g t$$

$$0 = 7.99 - 9.8 t$$

$$9.8 t = 7.99 \implies t = 0.82 \text{ sec.}$$

Which is $\frac{1}{2}$ of the motion.

To get the full motion, multiply we the time by 2.

$$0.82 \times 2 = 1.64 \text{ sec}$$

$$h = \frac{1}{2} g t^2$$

$$h = \frac{1}{2} \times 9.8 \times 0.82^2$$

$$= 0.5 \times 9.8 \times 0.82^2 = 3.29 \text{ m}$$

Horizontal motion formula.

Since acceleration on the horizontal motion is 0

$$X = V_{0x} t = 6.02 \times 1.62$$

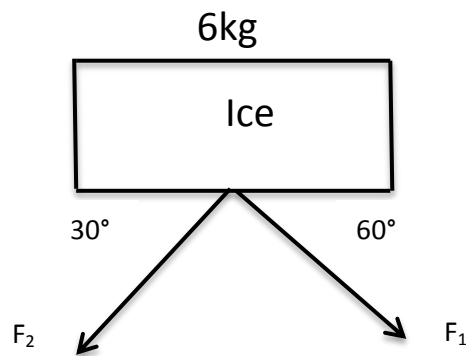
$$= 9.75 \text{ m}$$

-Newton's 2nd law-

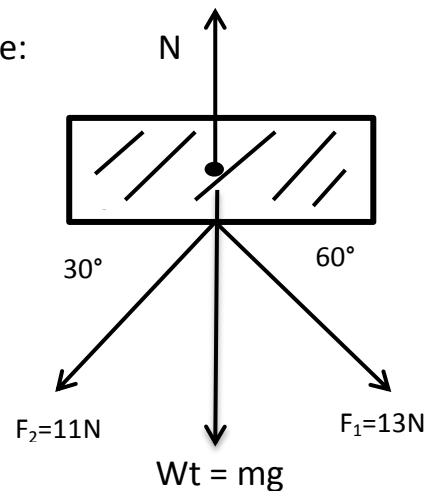
~Inclined plane~

Example1: A 6 kg block of ice is acted on by 2 forces \vec{F}_1 and \vec{F}_2 as shown in the diagram. If the magnitude of the forces are $F_1 = 13\text{N}$ and $F_2 = 11\text{N}$. find the acceleration of the ice and the normal force exerted on it by the table.

-Solution-



Forces acting on the block of ice:



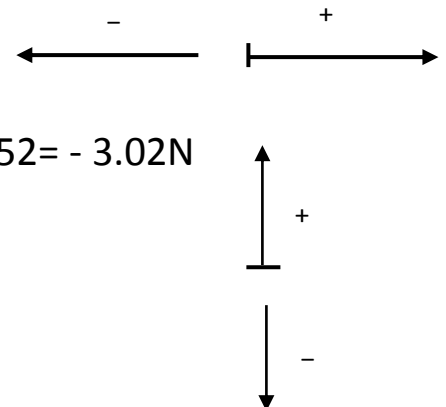
Separate the X and Y component of the forces.

X- components: $F_1 \cos 60^\circ - F_2 \cos 30^\circ$

$$= 13 \cos 60^\circ - 11 \cos 30^\circ = 6.5 - 9.52 = -3.02\text{N}$$

Y- components: $N - W - F_1 \sin 60^\circ - F_2 \sin 30^\circ$

$$\sum F_y = N - mg - 13 \sin 60^\circ - 11 \sin 30^\circ$$



$$= N - 6 \times 9.8 - 11.25 - 6.50 = N - 58.8 - 17.75 = N - 76.55$$

Newton's 2nd Law:

$$\sum F_x = ma_x$$

$$-3.02 = 6a_x \implies a_x = -0.5 \text{ m/s}^2$$

Since there is no vertical acceleration

$$a_y = 0$$

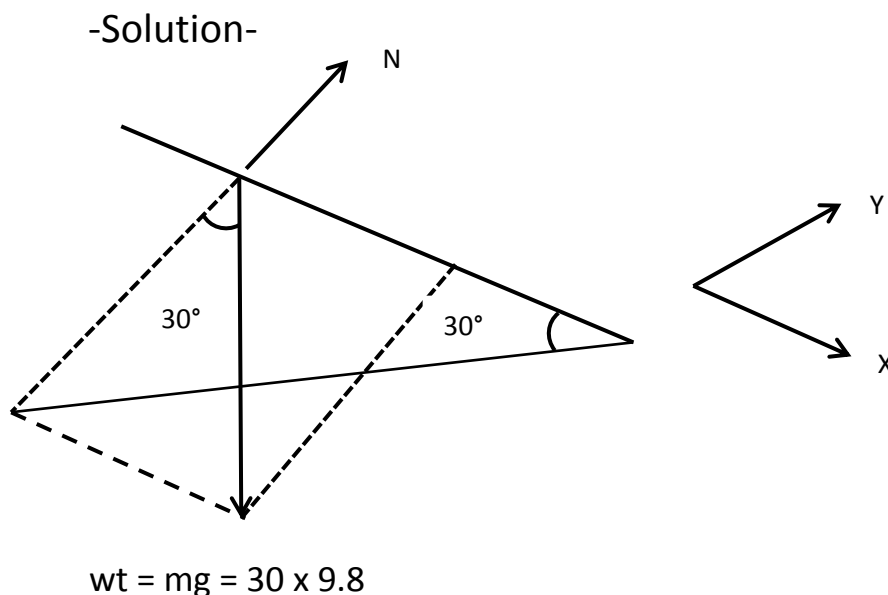
$$\sum F_y = ma_y = 6 \times 0 = 0$$

$$N - 76.55 = 0 \implies N = 76.55 \text{ N}$$

Example 2: A child of mass 30 kg rides on a toboggan, ice covered hill inclined at an angle 30° with respect to the horizontal

a, what is the acceleration of the child ?

b, what is the normal force exerted on the child by the toboggan ?



X- Components of the forces:

$$mg \sin 30^\circ \quad (\text{only})$$

$$= 30 \times 9.8 \times 0.5 = 147 \text{ N}$$

$$\sum F_x = m a_x$$

$$147 = 30a_x \longrightarrow a_x = 4.9 \text{ m/s}^2$$

Y- Components of the forces:

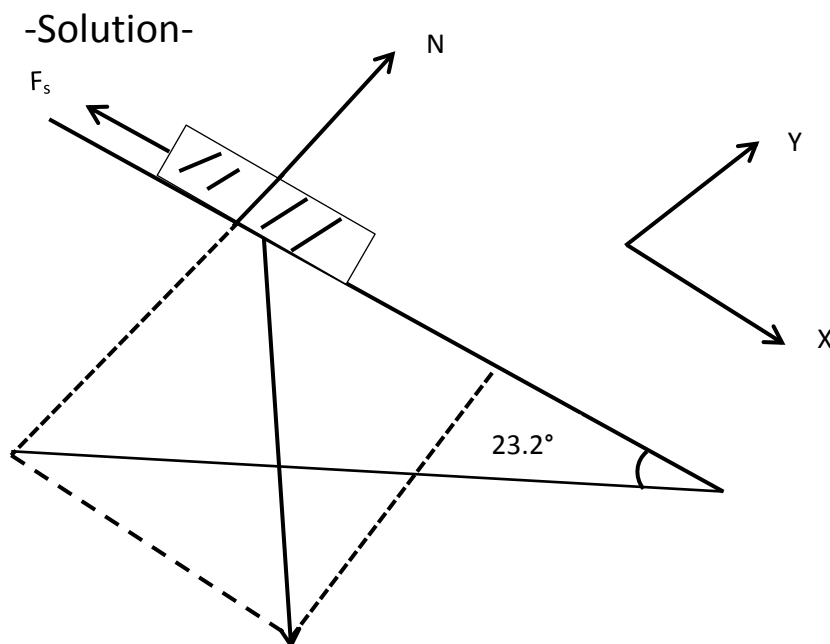
$$- mg \cos 30^\circ + N = m a_y \quad (a_y = 0)$$

$$- mg \cos 30^\circ + N = 0 \quad (\text{no vertical movement})$$

$$- 30 \times 9.8 \times 0.866 + N = 0$$

$$N = 254.61 \text{ N}$$

Example 3: A flatbed truck slowly tilts its bed upward to dispose of a 95 kg crate. But when the tilt angle exceeds 23.2° , the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?



$$wt = mg = 95 \times 9.8$$

X- Components of the forces:

$$mg \sin 23.2^\circ - F_s$$

$$\sum F_x = ma_x \quad (\text{Newton's 2}^{\text{nd}} \text{ law})$$

$$Mg \sin 23.2^\circ - F_s = ma_x \quad \text{but } a_x = 0$$

$$95 \times 9.8 \times 0.393 - F_s = 0 \implies F_s = 365.88 \text{ N}$$

Y- Components of the forces:

$$-mg \cos 23.2^\circ + N = -95 \times 9.8 \times 0.919 + N$$

$$\sum F_y = m a_y \quad \text{but } a_y = 0$$

$$N = 855.589 \text{ N}$$

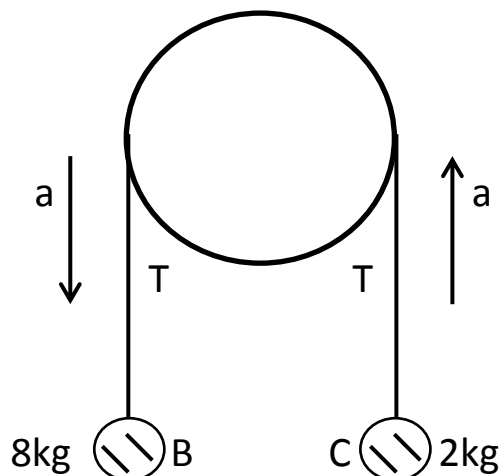
$$\text{Coefficient of static friction } \mu = \frac{F_s}{N}$$

$$= \frac{365.88}{855.589}$$

$$= 0.42$$

-Pulleys-

Example 1:



A light inextensible string passes over a smooth light pulley.

At each end of the string there is a particle. Particle B has a mass of 8 kg and particle C has a mass of 2 kg. The particles are released from rest with the string taut. Calculate the tension in the string and the acceleration of the masses.

-Solution-

Set up a (+) or (-) direction for the motion. (Upward is +), (downward is -). Work on mass B.

$$Wt = m \times g \text{ and } g = 9.8 \text{ m/s}^2$$

$$T - 8 \times 9.8 = -8 \times a$$

$$T - 78.4 = -8a$$

$$\text{Or, } T = -8a + 78.4$$

Work on mass C

$$-2 \times 9.8 + T = 2a$$

$$-19.6 + T = 2a$$

$$\text{Or, } T = 2a + 19.6$$

We have 2 equations:

$$T = -8a + 78.4$$

$$T = 2a + 19.6$$

Set them equal to each other.

$$-8a + 78.4 = 2a + 19.6$$

$$-10a = -58.8$$

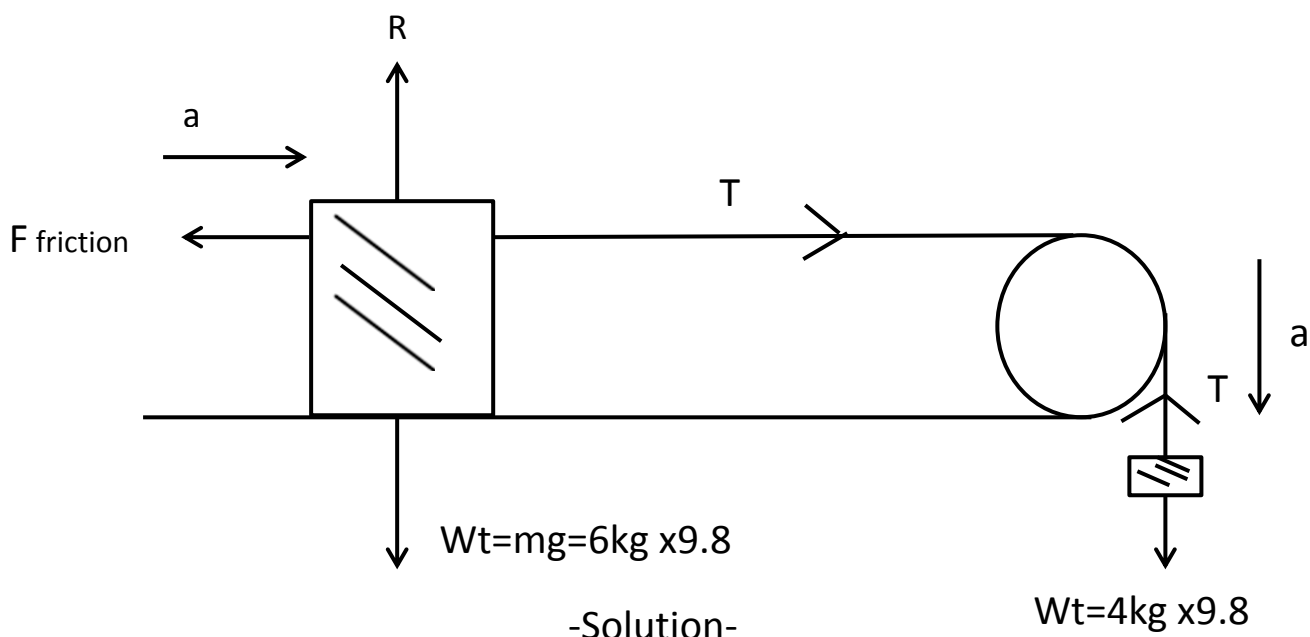
$$a = 5.88 \text{ m/s}^2$$

$$T = -8a + 78.4$$

$$T = -8(5.88) + 78.4 = 31.36 \text{ N}$$

Example 2:

A box, of mass 6 kg, is sliding along a rough horizontal table. It is connected to another box of mass = 4 kg, via a light inextensible string, which passes over a smooth light pulley. Given the coefficient of sliding friction between the table and the box is 0.3, what is the acceleration of each Particle and What is the tension in the string?



Right motion and upward motion (+) downward motion and left motion (-) .

Work on the 6 kg first:

$$F_{\text{friction}} = \mu \times R = 0.3 \times 6 \times 9.8 = 17.64 \text{ N.}$$

$$T - F_{\text{friction}} = ma$$

$$T - 17.64 = 6a \quad \text{or, } T = 6a + 17.64$$

Work on the 4 kg now:

$$T - Wt = -ma$$

$$T - 4 \times 9.8 = -4a$$

$$T = -4a + 39.2$$

Set T equal to each other.

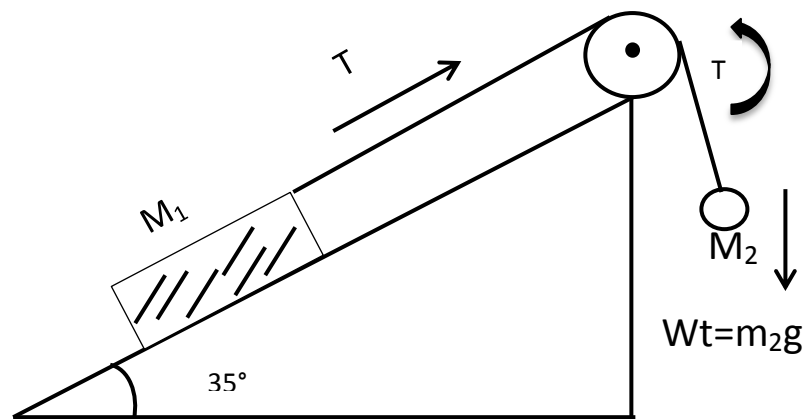
$$6a + 17.64 = -4a + 39.2$$

$$10a = 21.56 \implies a = 2.156 \text{ m/s}^2$$

To find T, use any equation

$$T = -4a + 39.2$$

$$T = -4(2.156) + 39.2 = 30.58 \text{ N}$$

Example 3:

A large weight $w_1 = 98 \text{ N}$ sits on a surface inclined at an angle of 35° from the horizontal. A smaller weight $w_2 = 58.8 \text{ N}$ is connected to the block by a string that goes up over a pulley. Suppose there is no friction. When the 2 weight are released, one will go up and the other will go down. Which weight goes down? what is the acceleration?

-Solution -

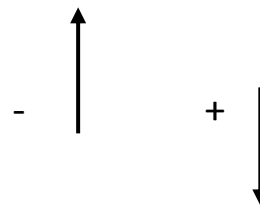
Work on m_2 :



$$- T + m_2 g = m_2 a$$

$$- T + 58.8 = 6a$$

$$\text{Solve for } T \implies T = 58.8 - 6a$$



$$m_2 = \frac{wt}{9.8} = \frac{58.8}{9.8} = 6 \text{ kg}$$

Work on m_1 :

$$T - m_1 g \sin \theta = m_1 a$$

$$m_1 = \frac{98}{9.8} = 10 \text{ N}$$

$$T - 98 \sin 35^\circ = 10 a$$

$$\text{Solve for } T \implies T = 10 a + 56.2$$

Set T equal to each other.

$$58.8 - 6 a = 10 a + 56.2$$

$$\implies a = 0.1618 \text{ m/s}^2$$

Since it is (+) the block moves downward. M_2 is more effective

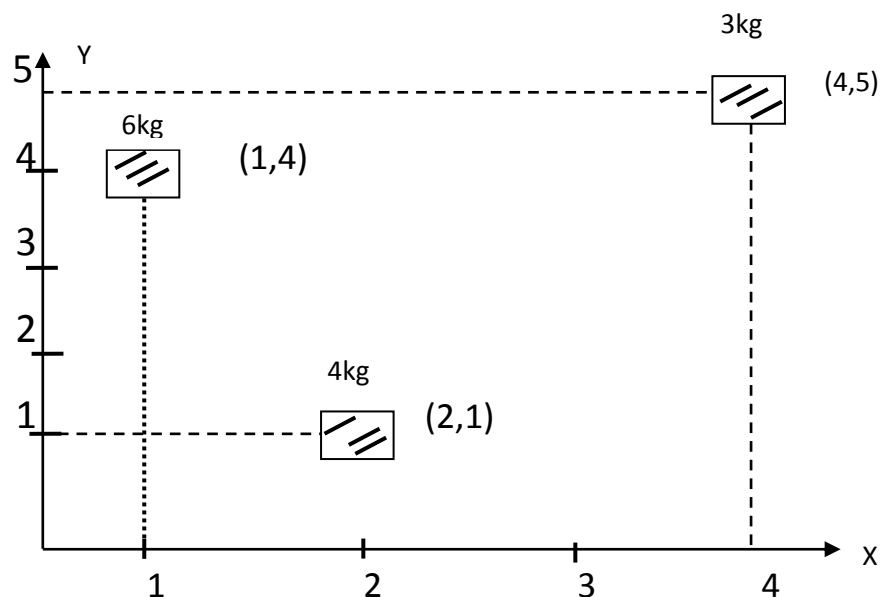
-Center of Mass-

Formulas:

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$$\bar{y} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

Example 1:



Find the coordinates of the center of mass.

-Solution-

Label the Coordinates of each mass:

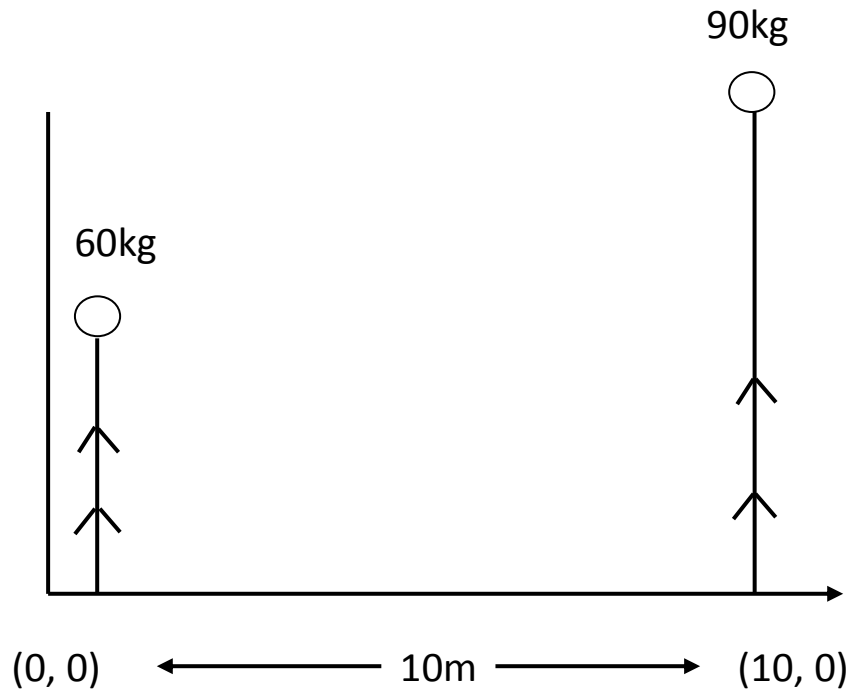
$$\bar{x} = \frac{4 \times 2 + 3 \times 4 + 6 \times 1}{4 + 6 + 6} = \frac{8 + 12 + 6}{16} = \frac{26}{16} = \frac{13}{8}$$

$$\bar{y} = \frac{4 \times 1 + 3 \times 5 + 6 \times 4}{16} = \frac{4 + 15 + 24}{16} = \frac{43}{16}$$

center of mass coordinates $(\frac{13}{8}, \frac{43}{16})$

Example 2: A 60 kg woman and a 90 kg man are standing 10 meter apart on frictionless ice .How far from the woman is the centers of mass of the system:

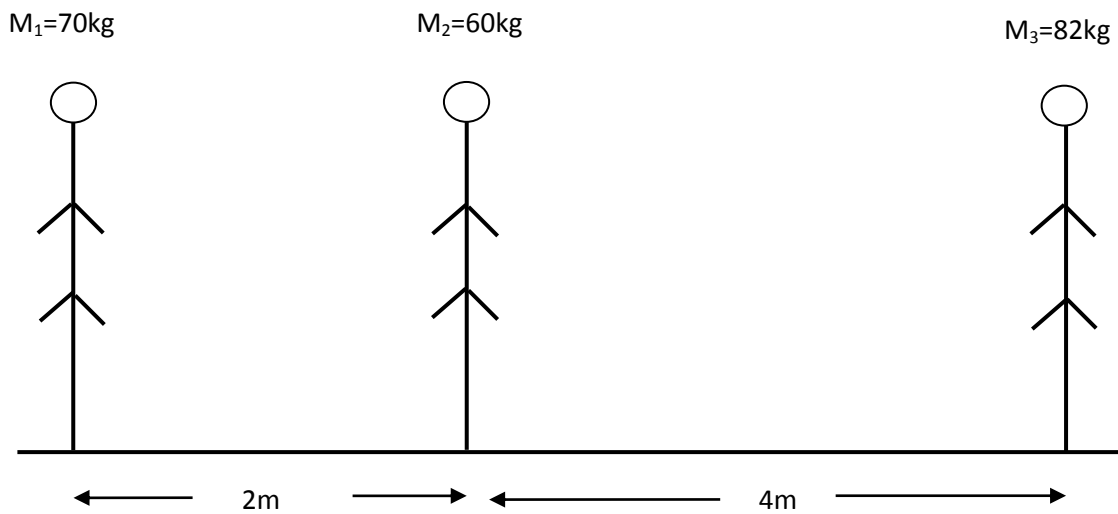
-Solution-



$$\bar{X} = \frac{60 \times 0 + 90 \times 10}{60 + 90} = \frac{900}{150} = 6 \text{ m}$$

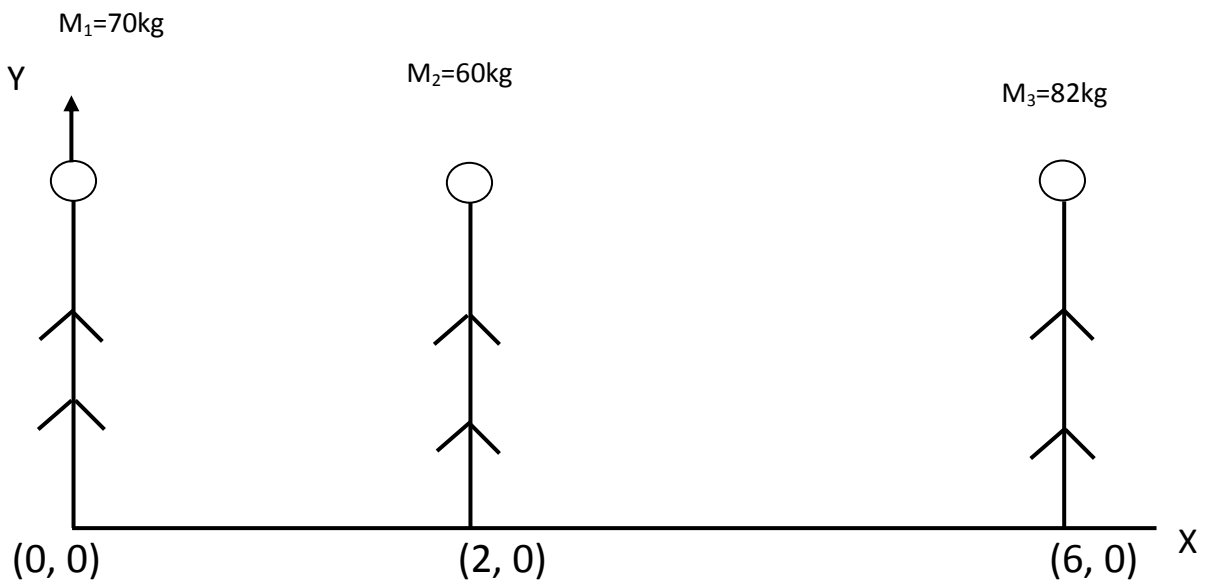
Disregard the y coordinate since everything is on the x-axis

Example 3: Three people are standing on the sidewalk as shown below.



Determine the coordinates of the center of mass for the 3 people.

-Solution-



$$\bar{X} = \frac{70 \times 0 + 60 \times 2 + 82 \times 6}{(70+60+82)} = 2.88\text{m}$$

-Law of conservation-

- Of Momentum –

Momentum = mass x velocity

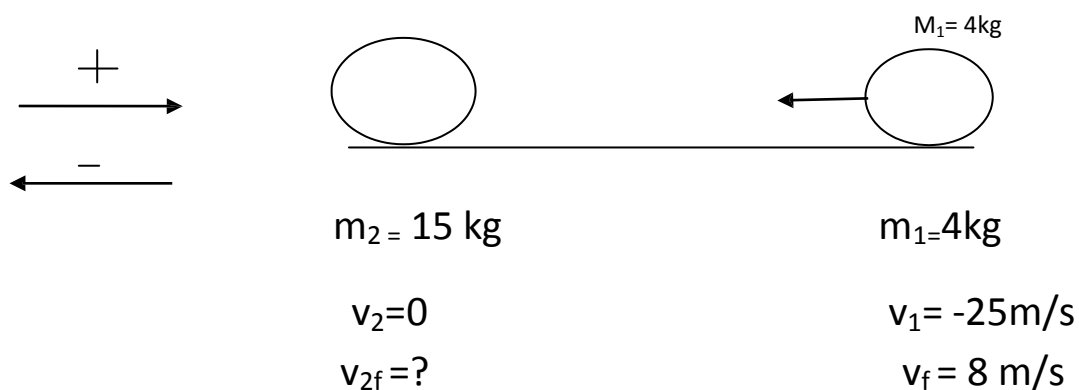
Momentum before collision = Momentum after collision.



$$m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$$

Example 1: A 4 kg ball traveling westward at -25 m/s hits a 15 kg ball at rest the 4 kg ball bounces east at 8m/s .what is the speed and direction of the 15 kg ball ?

- Solution-



$$m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$$

$$4 \times -25 + 15 \times 0 = 4 \times 8 + 15 \times v_{2f}$$

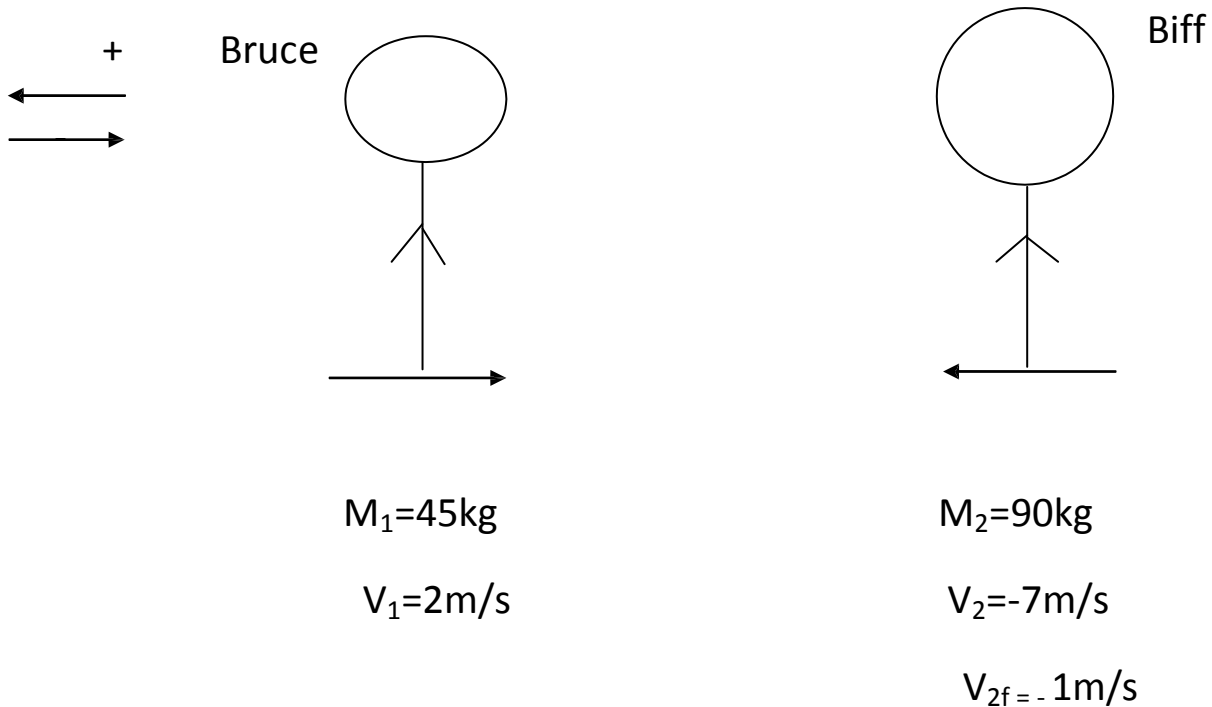
$$-100 = 32 + 15 v_{2f} \quad \longleftrightarrow \quad 15 v_{2f} = -132$$

$$v_{2f} = \frac{-132}{15}$$

$$= -8.8 \text{ m/s (west)}$$

Example 2: Running at 2m/s Bruce the 45 kg quarterback, collides with Biff the 90 kg tackle, who is traveling at -7m/s in the other direction. Upon collision, Biff continues to travel forward at 1m/s. How fast is Bruce knocked backward?

-Solution -



Law of conservation Of Momentum.

$$M_1 V_1 + M_2 V_2 = M_1 V_{1f} + M_2 V_{2f}$$

$$45 \times 2 + 90 \times -7 = 45 V_{1f} + 90 \times (-1)$$

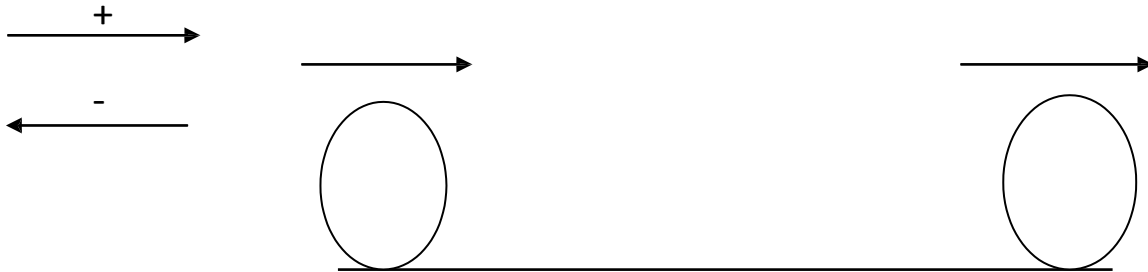
$$90 - 630 = 45 V_{1f} - 90$$

$$-540 = 45 V_{1f} - 90$$

$$45 V_{1f} = -450 \quad \overleftarrow{\hspace{1cm}} \quad V_{1f} = -10\text{m/s (left)}.$$

Example 3: A 2 kg ball traveling eastward at 5m/s hits a 5 kg ball at 1 m/s eastward . The 2 kg ball bounced west at 3 m/s. what is the speed and direction of the 5 kg ball?

-Solution-



$$m_1=2\text{Kg}$$

$$V_1=5\text{m/S}$$

$$V_{1f}=-3 \text{ m/S}$$

$$m_2=5\text{kg}$$

$$V_2=1\text{m/S}$$

$$V_{2f}=?$$

$$M_1 V_1 + M_2 V_2 = M_1 V_{1f} + M_2 V_{2f}$$

$$2 \times 5 + 5 \times 1 = 2 \times -3 + 5 V_{2f}$$

$$15 = -6 + 5 V_{2f}$$

$$V_{2f} = 4.2 \text{ m/S (to the right).}$$

Example 4: Miguel, the 72 kg bull fighter, runs towards an angry bull at a speed of 4 m/s. the 550 kg bull charges Miguel at -12 m/s and Miguel jumps on his back. What is the new velocity of Miguel and the bull?

-Solution-

$$\begin{aligned}
 & \xrightarrow{+} \quad M_1 V_1 + M_2 V_2 = (M_1 + M_2) V_f \\
 \xleftarrow{-} \quad & 72 \times 4 + 550 \times -12 = (72 + 550) V_f \\
 & 288 - 6600 = 622 V_f \\
 & -6312 = 622 V_f \quad \longleftrightarrow \quad V_f = -10.14 \text{ m/s (left)}
 \end{aligned}$$

Example 5: two balls hit head on as shown .what is the final velocity of the second ball if the first one's final velocity is -1.5 m/s

-Solution-

Given:



$$m_1 = 1.50 \text{ kg}$$

$$m_2 = 1.85 \text{ kg}$$

$$v_1 = 2.30 \text{ m/s}$$

$$v_2 = -1.30 \text{ m/s}$$

Law of conservation Of Momentum.

$$M_1 V_1 + M_2 V_2 = M_1 V_{1f} + M_2 V_{2f}$$

$$1.50 \times 2.30 + 1.85 \times -1.30 = 1.50 \times -1.5 + 1.85 V_{2f}$$

$$3.45 - 2.405 = -2.25 + 1.85 V_{2f}$$

$$1.045 = -2.25 + 1.85 V_{2f}$$

$$3.295 = 1.85 V_{2f}$$

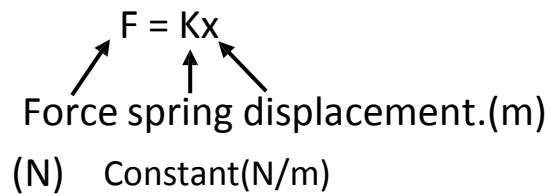
$$V_{2f} = 1.785 \text{ m/s} \quad (\text{to the right})$$

-Hook's Law-

Example 1: A spring with a spring constant of 10 N/m is stretched 0.5 m. How much force is applied to the spring?

-Solution-

$$F = Kx$$



Force spring displacement.(m)
 (N) Constant(N/m)

$$F = K x = 10 \times 0.5 = 5 \text{ N}$$

Example 2: The force required to stretch a spring varies from 0N to 65N as we stretch the spring moving one end 6.3cm from its unstressed position. Find the spring constant of the spring.

-Solution-

$$F = K x$$

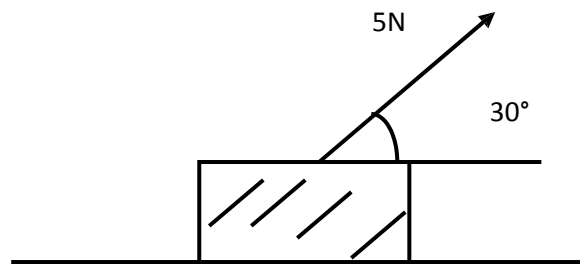
$$65 = K \times 0.063$$

$$K = 65/0.063 = 1032 \text{ N/m}$$

Potential Energy, Kinetic energy and work

Example 1: A box is pulled along the floor a distance of 10 m and the force pulling it has a magnitude of 5N and is directed 30° above the horizontal. Find the work done on the box.

-Solution-



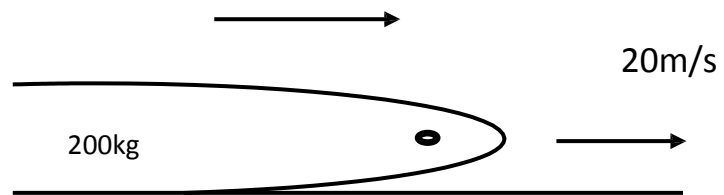
Horizontal component of the force

$$= 5 \cos 30^\circ = 4.33 \text{ N}$$

$$W = F \times d = 4.33 \times 10 = 43.3 \text{ N}$$

Example 2: A train car with a mass of 200 kg is traveling At 20 m/s. How much force must be exerted on the brakes to stop the train car in a distance of 10m?

-Solution-



Work_{net} = change in kinetic Energy

$$\begin{aligned} W_{\text{net}} &= \frac{1}{2} mv_f^2 - \frac{1}{2} mv_0^2 \\ &= (0) - 0.5 \times 200 \times 20^2 \\ &= - 40000 \text{ Joules} \end{aligned}$$

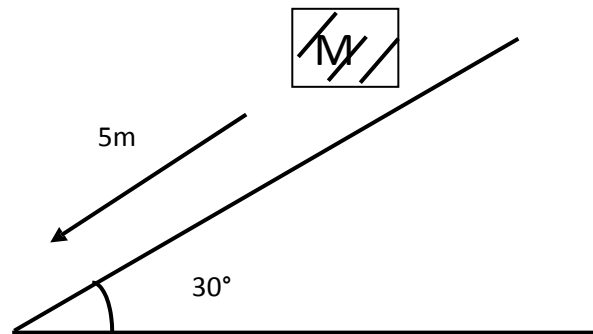
But W_{net} is also = $F \times d$

$$F \times 10 = - 40000$$

$$F = - 4000 \text{ N (opposite to the direction of displacement)}$$

Example 3: A block of mass m is placed on a frictionless Plane inclined at a 30° angle above the horizontal. It is released from rest and allowed to slide $5m$ down the Plane. What is its final velocity?

-Solution-



Using the law of conservation of Energy:

Initial potential + Kinetic energy = final potential + final Kinetic energy.

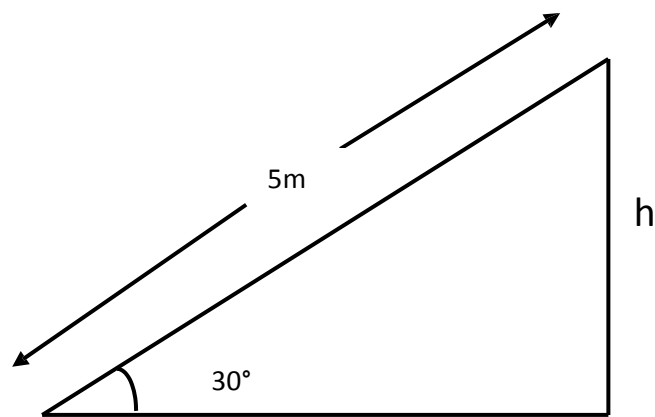
$$\text{Or } K - E_i + P - E_i = K - E_f + P - E_f$$

Since the initial velocity = $0 \Rightarrow K - E_i = 0$ and since the box slides all the way to the end \Rightarrow final height = 0

$$\text{Or, } P - E_f = 0$$

$$P - E_i = K - E_f$$

$$\text{Or } mgh = \frac{1}{2} mv_f^2$$



$$\sin 30^\circ = \frac{h}{5} \text{ or } h = 5 \sin 30^\circ = 2.5\text{m}$$

$$m \times 9.8 \times 2.5 = 0.5 \times m \times v_f^2$$

Dividing both sides by m.

$$9.8 \times 2.5 = 0.5 V_f^2$$

$$24.5 = 0.5 V_f^2$$

$$V_f^2 = 49 \implies V_f = 7 \text{ m/s.}$$

Angular Displacement, Velocity, and acceleration

Angular Displacement = S

= $r\theta$, r is the

Radius and θ is the angle in radians

It means how far is the arc from A to B

Angular Velocity: $V = WR$ or $w = \frac{v}{r}$

Angular Velocity = $\frac{\text{linear Velocity}}{\text{radius}}$

How many turns or spins or revolution /sec

Angular acceleration: $\bar{a} = \frac{w}{t} = \frac{\text{angular Velocity}}{\text{time}}$

In other words, by how much did you increase or decrease the speed of the spins in 1 second unit is: Rev/s^2

More Formulas:

$$W_f = W_0 + \bar{a} t$$

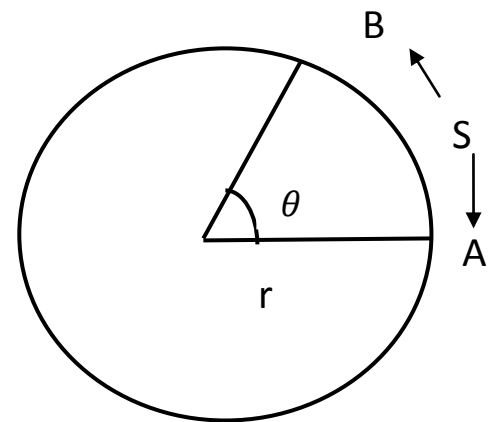
$$\theta = \frac{1}{2} (W_f + W_0) t$$

$$\theta = w_0 t + \frac{1}{2} \bar{a} t^2$$

$$W_f^2 = W_0^2 + 2 \bar{a} \theta$$

$$F_c = m w^2 R \quad \text{Or} = \frac{mv^2}{R}$$

$$a_c = w^2 R \quad \text{or} \quad \frac{v^2}{R}$$



Example 1: A skater initially turning at 3 rev/sec down With constant angular deceleration and stops in 4 sec. Find her angular deceleration and the number of revolutions she makes before stopping.

-Solution-

Given: 3rev/sec is w_0

$w_f = 0$ (she stopped)

$t = 4\text{sec}$

Angular deceleration $= \bar{a}$

The formula that involves what's given is

$$w_f = w_0 + \bar{a} t$$

$$0 = 3 + 4\bar{a}$$

$$4\bar{a} = -3 \implies \bar{a} = -3/4 \text{ or } -0.75\text{srev/s}^2$$

of revolution is θ

To find θ , we use the formula that involves what's given.

$$\begin{aligned} \theta &= w_0 t + \frac{1}{2} \bar{a} t^2 \\ &= 3 \times 4 + 0.5 \times -0.75 \times 4^2 \\ &= 12 - 6 = 6 \text{ revolutions.} \end{aligned}$$

Example 2: when a bowling ball is first released, It slides down the alley before it starts rolling. If it takes 1.2 seconds for a bowling ball to attain an angular Velocity of 6rev/sec, determine, the average angular acceleration of the bowling ball.

$$\text{Given: } W_0 = 0$$

$$W_f = 6\text{rev/sec}$$

$$t = 1.2 \text{ sec}$$

The formula that involves what's given is

$$W_f = W_0 + \bar{a} \cdot t$$

$$6 = 0 + \bar{a} \times 1.2$$

$$6 = 1.2\bar{a}$$

$$\bar{a} = 5\text{rev/s}^2$$

Example 3: what is the magnitude of the angular Velocity of the second hand clock? What is its direction as you view the clock hanging vertically? What is the magnitude of the angular acceleration of the second hand?

-Solution-

The second hand rotates 1 revolution in 1 min or 2π radians in 1 minute but since the angular Velocity is in radians /sec $\frac{2\pi}{60} = \frac{2 \times 3.14}{60} = 0.104$

Its direction is perpendicular to the face of the clock, pointing into the face of the clock. The average angular acceleration is 0 because the angular Velocity is constant.

Example 4: what is the angular speed, in radians per second, of a, the Earth in its orbit around the sun. b, and the moon in its orbit around the earth.

-Solution-

The earth orbits the sun once every year or 2π in 1 year.

$$\frac{2\pi \text{ rad}}{1 \text{ year}} \times \frac{1 \text{ year}}{365 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= \frac{2 \times 3.14}{31536000} = 0.0000002 \text{ rad/sec}$$

b, The moon orbits the earth once every 27.3 days or 2π in 27.3 days.

$$\frac{2\pi \text{ rad}}{27.3 \text{ d}} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{6.28}{2358720}$$

$$= 0.00000266 \text{ rad/sec}$$

Example 5: Calculate the centripetal acceleration and force acting on an airplane of mass 1500 kg turning on a circle 400 m radius at a velocity of 300 m/s

-Solution-

Given: $m = 1500\text{kg}$, $R = 400\text{m}$, $V = 300\text{ m/s}$

The formula that involves what's given is

$$a_c = \frac{v^2}{r} = \frac{300^2}{400} = \frac{300 \times 300}{400} = 225\text{m/s}^2$$

$$F_c = m \times a_c = 1500 \times 225 = 337500\text{ N}$$

Example 6: calculate the centripetal force acting on a small mass of 0.5Kg rotating at 1500 revolution / min on a radius of 0.3m

-Solution-

Given: 1 revolution = 2π radians

ω = angular velocity is in radians / sec

$$\omega = 1500 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{1500 \times 2 \times 3.14}{60} = 157\text{rad/sec}$$

$$a_c = \omega^2 R = 157^2 \times 0.3 = 7395\text{ m/s}^2$$

$$F_c = m \times a = 0.50 \times 7395 = 3697\text{ N}$$

Example 7: A car travels a curve of radius 200m at 14m/s on a force of 965 N.
what is the weight of the Van?

-Solution-

Given: $V = 14 \text{ m/s}$

$R = 200\text{m}$

$F = 965 \text{ N}$

The formula that involves what's given is:

$$F = \frac{mv^2}{r^2}$$

$$965 = m \times 14^2/200$$

$$\frac{965}{1} = \frac{196m}{200} \quad \text{cross multiply}$$

$$196 m = 193000$$

$$m = 985 \text{ kg}$$

Angular Momentum and Torque

-Formulas-

Moment of inertia I

$$K.E = \frac{1}{2} I \omega^2$$

(ω is the angular velocity)

$$\text{Torque } T = I \alpha$$

(α is the angular acceleration)

$$\text{Angular Momentum} = L = I \omega$$

-Angular Momentum and Torque-

Example 1: A Carousel – a horizontal rotating platform – of radius initially at rest, and then begins to accelerate constantly until it has reached an angular velocity W after 2 complete revolutions. What is the angular acceleration of the carousel during this time?

-Solution-

$$W^2 = W_0^2 + 2 \bar{a} \theta$$

$$\theta = 2 \text{ revolutions} = 2 \times 2\pi = 4\pi$$

$$W_0 = 0$$

$$W^2 = 2 \bar{a} \times 4\pi$$

$$W^2 = 8 \pi \bar{a} \implies \bar{a} = \frac{W^2}{8\pi}$$

Example 2: A horizontally – mounted disk with moment of inertia. I spins about a frictionless axle. At time $t = 0$ the initial angular speed of disc is w . A constant torque T is applied to the disc, causing it to come to a halt in time t . How much power is required to dissipate the wheel's energy during this time ?

-Solution-

$$P = \frac{W}{t}$$

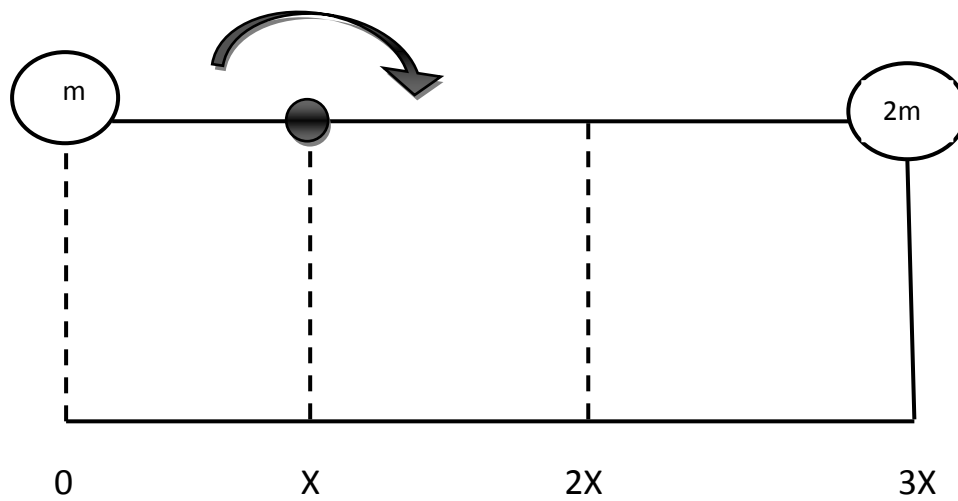
$W = \text{change in kinetic energy} = \Delta K. E.$

$$K.E = \frac{1}{2} I w^2$$

$$W = \frac{1}{2} I w^2 - \frac{1}{2} I w_0^2 \quad \text{since it stopped } w = 0$$

$$W = -\frac{1}{2} I w_0^2$$

$$\text{But } p = \frac{W}{t} = \frac{I w_0^2}{2t}$$

Example 3

A solid sphere of mass m is fastened to another sphere of mass $2m$ by a thin rod with a length of $3x$. The spheres have negligible size and the rod has negligible mass. What is the moment of inertia of the system of spheres as the rod is rotated about the point located at position x ?

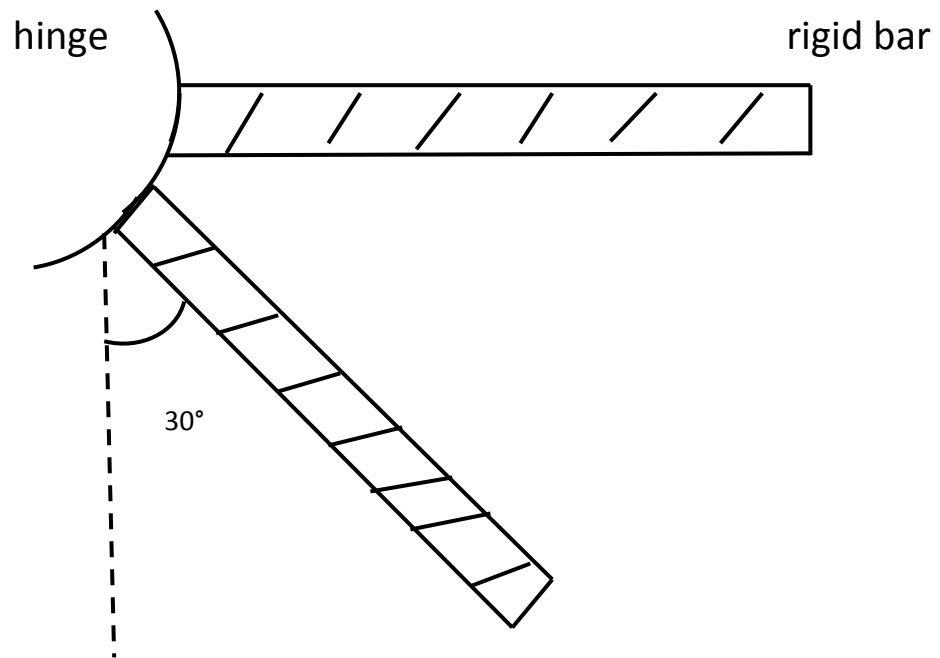
-Solution-

$$I = \text{moment of inertia} = \text{sum of } mr^2$$

$$I = m (x)^2 + 2m (2x)^2$$

$$= mx^2 + 2m (4x^2) = 9mx^2$$

Example 4: A rigid bar with a mass M and Length L is free to rotate about a frictionless hinge at a wall. The bar has a moment of inertia $I = \frac{1}{3} ML^2$ about the hinge and is released from rest. When it is in a horizontal position as shown. What is the instantaneous angular acceleration. When the bar has swung down so that it makes an angle of 30° to the vertical.



-Solution-

$$\text{Torque} = I \alpha$$

$$r = \frac{L}{2}$$

$$r \times F = \frac{1}{3} ML^2 \alpha$$

$$F = mg \sin 30^\circ$$

$$0.5 mg \frac{L}{2} = \frac{1}{3} mL^2 \alpha$$

$$= 0.5mg$$

$$\text{Cross multiply} = 1.5 mg L = 2 mL^2 \alpha \implies \alpha = \frac{1.5 mgL}{2 mL^2}$$

$$= \frac{0.75 g}{L}$$

Newton's law of Universal

-Gravitation-

$$F = G \frac{m_1 m_2}{r^2}$$

F = gravitation force (N)

M_1 and m_2 are masses in kg

r = is the distance between masses in meters

G = is constant = $6.673 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

Example 1: communication satellites orbit the earth at a height of 36000 km. How far is this from the center of the earth? If such a Satellite has a mass of 250 kg. What is the force of attraction on it from the earth?

-Solution-

Radius of the earth = $6.4 \times 10^6 \text{ m}$.

$h = 36000 \text{ km} = 36000 \times 1000 = 36000000$

$= 3.6 \times 10^7 \text{ m}$

Satellite is $(6.4 \times 10^6 + 3.6 \times 10^7) \text{ m}$ away from earth or $4.24 \times 10^7 \text{ m}$

G is constant = 6.673×10^{-11}

$m_1 = 250 \text{ kg}$

$m_2 = \text{mass of earth} = 6 \times 10^{24} \text{ kg}$

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 250 \times 6 \times 10^{24}}{(4.24 \times 10^7)^2} = 56\text{N}$$

Example 2: At what distance apart would 2 equal masses of 150 kg need to be placed for the force between them to be 2×10^{-5} N?

-Solution-

$$r = ?$$

$$m_1 = 150 \text{ kg}$$

$$m_2 = 150 \text{ kg}$$

$$F = 2 \times 10^{-5} \text{ N}$$

$$G = 6.67 \times 10^{-11}$$

$$F = \frac{Gm_1m_2}{r^2}$$

$$\frac{2 \times 10^{-5}}{1} = \frac{6.67 \times 10^{-11} \times 150 \times 150}{r^2}$$

Cross multiply:

$$2 \times 10^{-5} r^2 = 0.0000015$$

$$0.00002 r^2 = 0.0000015$$

$$r^2 = 0.075$$

$$r = \sqrt{0.075} \text{ m} = 0.27\text{m}$$

Example 3: The average force of attraction on the moon from the sun is 4.4×10^{20} N. if we assume that The distance from the moon to the sun is 1.5×10^{11} m, what value of mass does this give for the moon?

-Solution-

$$F = 4.4 \times 10^{20} \text{ N}$$

$$m_1 = \text{mass of the sun} = 2 \times 10^{30} \text{ kg}$$

$$m_2 = \text{mass of the moon} = ?$$

$$r = 1.5 \times 10^{11} \text{ m}$$

$$G = 6.67 \times 10^{-11}$$

$$F = \frac{Gm_1m_2}{r^2}$$

$$\frac{4.4 \times 10^{20}}{1} = \frac{6.67 \times 10^{-11} \times 2 \times 10^{30} \times m_2}{(1.5 \times 10^{11})^2}$$

Cross multiply:

$$4.4 \times 10^{20} \times (1.5 \times 10^{11})^2 = 6.67 \times 10^{-11} \times 2 \times 10^{30} \times m_2$$

$$9.9 \times 10^{42} = 1.334 \times 10^{20} m_2$$

$$m_2 = 7.42 \times 10^{22} \text{ kg}$$

-Vibrations and Waves –

-Formulas-

$$PE = \frac{1}{2} kx^2$$

K is the spring constant, X is the stretch

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T: period and l is the length of the pendulum.

$$\lambda = \frac{v}{f}$$

λ is the wavelength in m.

$$f = \frac{1}{\lambda}$$

f is the frequency in Hz.

$$v = \lambda f$$

v is the velocity in m/s.

Example 1: A spring with a spring constant of 144 N/m is compressed by a distance of 16.5 cm. How much elastic potential energy is stored in the spring?

-Solution-

$$P.E = \frac{1}{2} kx^2$$

$$X = 16.5 \text{ cm} \div 100 = 0.165 \text{ m}$$

$$P. E = \frac{1}{2} \times 144 \times 0.165^2 = 1.96 \text{ joules}$$

Example 2: How long must a pendulum be on the moon, where $g = 1.6 \text{ m/s}^2$ to have a period of 2 seconds?

-Solution-

$$T = 2 \pi \sqrt{\frac{l}{g}}$$

$$2 = 2 \times 3.14 \times \sqrt{\frac{l}{1.6}}$$

$$2 = 6.28 \times \sqrt{\frac{l}{1.6}}$$

Divide both sides by 6.28

$$0.318 = \sqrt{\frac{l}{g}}$$

square both sides

$$0.1014 = \frac{l}{1.6}$$

cross multiply:

$$L = 0.16 \text{ m}$$

Example 3: The sound wave has a frequency of 436 Hz. What is the period of the wave?

-Solution-

$$T = \frac{1}{f} = \frac{1}{436} = 2.29 \times 10^{-3} \text{ second}$$

b, what is the wavelength?

-Solution-

Speed of sound = 343 m/ s

$$\lambda = \frac{v}{f} = \frac{343}{436} = 0.787 \text{ m}$$

Example 4: Suppose you hold a 1 meter bar in your hand and hit its end with a hammer, first in a direction parallel to its length, and second in a direction at right angle to its length. describe the waves produced in the 2 cases.

-Solution-

In the first case, longitudinal waves; in the second case, it is transverse waves.

Example 5: when a 225 g mass is hung from a spring, the spring stretches 9.4cm. The spring and mass then are pulled 8 cm from this new equilibrium position and released. Find the spring constant of the spring and the maximum speed of the mass?

-Solution-

$$M = 225 \text{ g} = 225 \div 1000 = 0.225 \text{ kg}$$

$$F = Kx$$

F is the weight

$$= mg = 0.225 \times 9.81 = 2.21 \text{ N}$$

$$F = Kx$$

$$X = 9.4 \text{ cm} = 0.094 \text{ m}$$

$$2.21 = Kx$$

$$2.21 = K \times 0.094 \Rightarrow k = 23 \text{ N/m}$$

Maximum velocity occurs when the mass passes through the equilibrium point, where all the energy is kinetic.

$$P.E_{SP} = K E_{mass}$$

$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

Or

$$kx^2 = mv^2$$

$$23 \times (0.08)^2 = 0.225 \times v^2$$

$$0.1472 = 0.225 v^2$$

$$v^2 = 0.654$$

$$v = \sqrt{0.654}$$

$$= 0.808 \text{ m/s}$$

~ Sound waves ~

Example 1: You are in an auto traveling at 25m/s toward a pole mounted warning siren. If the siren's frequency is 365 Hz. What frequency do you hear?
Speed of sound = 343m/s

-Solution -

$$V = 343 \text{ m/s}$$

$$V_s = 0$$

$$f_s = 365 \text{ Hz}$$

$$V_d = -25 \text{ m/s}$$

$$\begin{aligned} f_d &= f_s \frac{(V-V_d)}{(V-V_s)} \\ &= 365 \left(\frac{343+25}{343} \right) \\ &= 392 \text{ Hz} \end{aligned}$$

Example 2: You are in an auto traveling at 55 miles/ hr (24.6 m / s). A second auto is moving towards you at the same speed. Its horn is sounding at 475 Hz. What frequency do you hear?

-Solution -

$$V = 343 \text{ m/s}$$

$$f_s = 475 \text{ Hz}$$

$$V_s = +24.6 \text{ m/s}$$

$$V_d = - 24.6 \text{ m/s}$$

$$\begin{aligned} f_d &= f_s \frac{(V-V_d)}{(V-V_s)} \\ &= 475 \left(\frac{343+24.6}{343-24.6} \right) \\ &= 548 \text{ Hz} \end{aligned}$$

-Coulomb's law-

1 Coulomb is equal to the charge of 6.25×10^{18} electrons or protons. the charge of a single electron is -1.60×10^{-19} C. the charge of a single proton $+1.60 \times 10^{-19}$ C the coulomb is a Large amount of charge so we use either micro coulombs or milli coulombs,

$$1\text{mc}=10^{-3}\text{c}$$

$$1\mu\text{c}=10^{-6}\text{c}$$

$$\text{Coulomb's } F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$$

F is the force between the 2 charges q_1 and q_2 r is the distance between the charges $\frac{1}{4\pi\epsilon}$ Is the universal gravitational constant

$$=8.99 \times 10^9 \text{ nm}^2 / \text{c}^2$$

Example1: Two point's charges are 5m apart the charges are 0.020C and 0.030C what is the force between? It is attractive or repulsive?

-Solution-

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ (it is constant)}$$

$$q_1 = 0.020 \text{ C}$$

$$q_2 = 0.030 \text{ C}$$

$$r = 5 \text{ m}$$

$$F = 8.99 \times 10^9 \times \frac{0.020 \times 0.030}{5^2} = 2.2 \times 10^5 \text{ N}$$

Since both charges are (+) it is repulsive.

Example2: A force of $1.6 \times 10^{-3} \text{ N}$ exists between 2 charges $1.3 \mu\text{C}$ and $3.5 \mu\text{C}$. How far apart are they?

-Solution-

$$q_1 = 1.3 \mu\text{C} = 1.3 \times 10^{-6} \text{ C}$$

$$q_2 = 3.5 \mu\text{C} = 3.5 \times 10^{-6} \text{ C}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$F = 8.99 \times 10^9 \times \frac{1.3 \times 10^{-6} \times 3.5 \times 10^{-6}}{r^2}$$

$$\frac{1.6 \times 10^{-3}}{1} = \frac{8.99 \times 10^9 \times 1.3 \times 10^{-6} \times 3.5 \times 10^{-6}}{r^2}$$

Cross multiply:

$$1.16 \times 10^{-3} r^2 = 0.0409$$

$$r^2 = 25.56$$

$$r = \sqrt{25.56}$$

$$= 5.05 \text{ m}$$

Example 3: $q_1=6 \times 10^{-9} \text{C}$, $q_2=2 \times 10^{-9} \text{C}$,

$q_3=5 \times 10^{-9} \text{C}$.

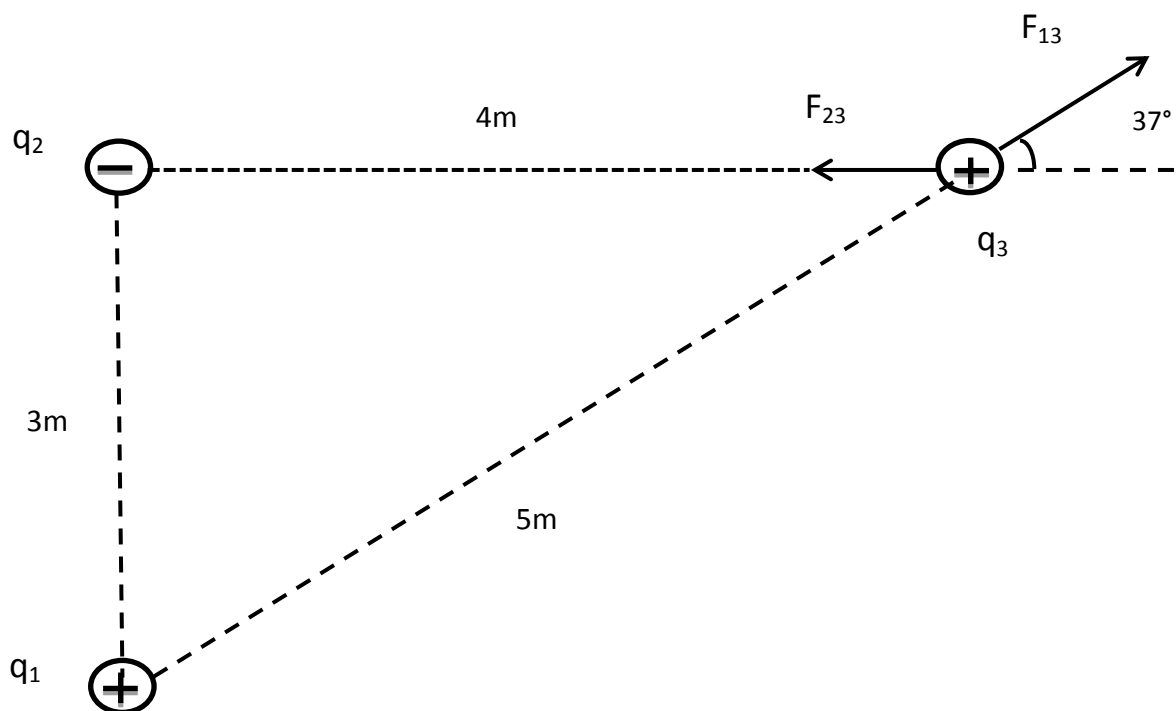
q_1 is 3m from q_2 and is (+)

q_2 is 4m from q_3 and is (-)

q_3 is (+)

What is the net force acting on q_3 ?

-Solution-



Net force on $q_3 = F_{23} + F_{13}$

$$F_{23} = 8.99 \times 10^9 \times \frac{q_1 q_3}{r^2}$$

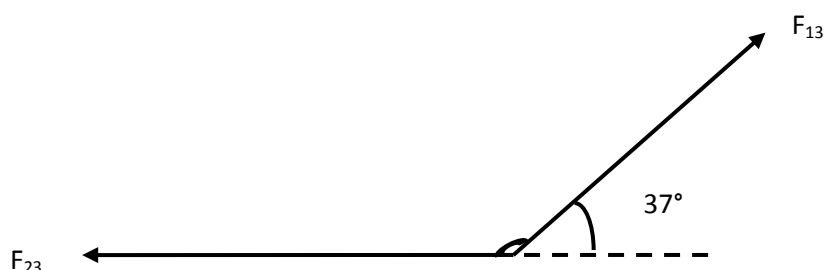
$$= 8.99 \times 10^9 \times \frac{2 \times 10^{-9} \times 5 \times 10^{-9}}{4^2}$$

$$= 5.62 \times 10^{-9} \text{ N}$$

$$F_{13} = 8.99 \times 10^9 \times \frac{q_1 q_3}{r^2}$$

$$8.99 \times 10^9 \times \frac{5 \times 10^{-9} \times 6 \times 10^{-9}}{5^2}$$

$$= 1.08 \times 10^{-8} \text{ N}$$



Break down F_{13} in to X and Y component:

$$F_{13}(\text{x component}) = 1.08 \times 10^{-8} \cos 37^\circ$$

$$= 8 \times 10^{-9} \text{ N}$$

$$F_{13}(\text{y component}) = 1.08 \times 10^{-8} \sin 37^\circ$$

$$= 6 \times 10^{-9} \text{ n}$$

F_{23} has only an x component

$$= -5.62 \times 10^{-9} \text{ (going left)}$$

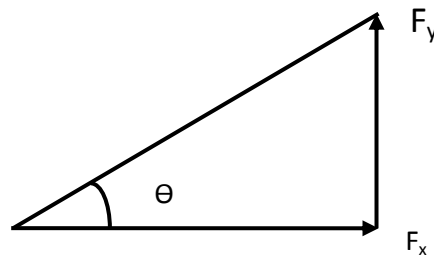
Add the x components together:

$$8 \times 10^{-9} - 5.62 \times 10^{-9} = 2 \times 10^{-9} \text{ N}$$

The total Y component = 6×10^{-9} N

$$\begin{aligned} F_{\text{net}} &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{(2 \times 10^{-9})^2 + (6 \times 10^{-9})^2} \\ &= 6 \times 10^{-9} \text{ N} \end{aligned}$$

Direction of the resultant force



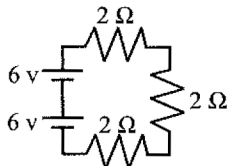
$$\begin{aligned} \theta &= 2^{\text{nd}} \tan\left(\frac{F_y}{F_x}\right) \\ &= 2^{\text{nd}} \tan\left(\frac{6 \times 10^{-9}}{2 \times 10^{-9}}\right) \\ &= 71^\circ \end{aligned}$$

V, R, and I in Series Circuits

Total Voltage (V_T)

If the batteries are in series (in a line) then **add them together** to find the total voltage (V_T).

$$V_T = V_1 + V_2 + V_3 + \dots$$

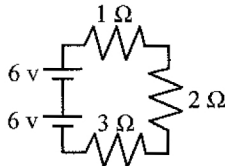


$$V_T = 6\text{ v} + 6\text{ v} = 12\text{ v}$$

Total Resistance (R_T)

If the resistors are in series then **add them together** to find the total resistance (R_T).

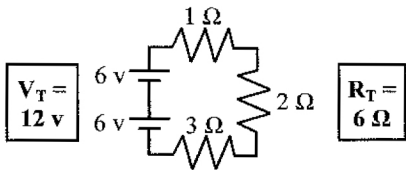
$$R_T = R_1 + R_2 + R_3 + \dots$$



$$R_T = 1\ \Omega + 2\ \Omega + 3\ \Omega = 6\ \Omega$$

Total Current (I_T)

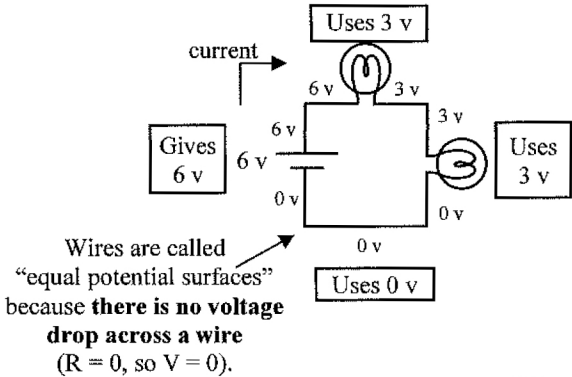
Use **Ohm's Law** to calculate the total current from V_T and R_T .



$$I = \frac{V}{R} = \frac{12\text{ v}}{6\ \Omega} = 2\text{ A}$$

Voltage Drop

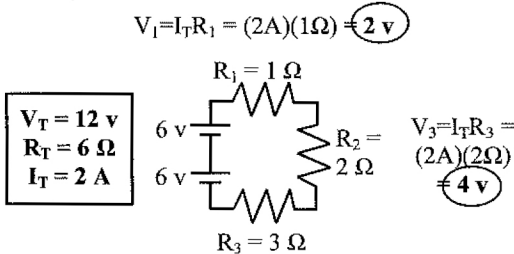
Each resistor in a series circuit "uses" part of the energy of the circuit, reducing the voltage. Eventually the voltage is back to zero at the negative side of the battery. Then the battery energizes the electrons again.



A circuit uses up all the voltage given by the batteries. Batteries give voltage: circuits use voltage. The voltage at the negative end of the batteries is always zero!

Voltage Across a Resistor

Calculating Voltage over a particular resistor:
 1) find the total current;
 2) use Ohm's Law for that resistor.



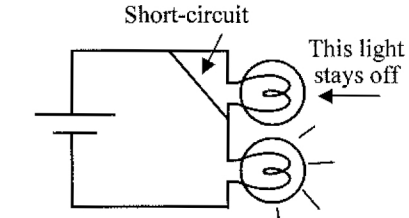
Notice that each resistor uses up part of the voltage and that all of the individual voltage drops equal V_T .

$$V_{RX} = I_T R_X, \text{ where } R_X \text{ is a particular resistor.}$$

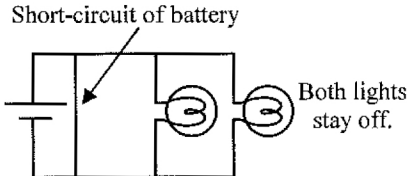
Short Circuits

Electricity always chooses the path of least resistance. Since wires have virtually no resistance, electricity will go thru a wire instead of a device or circuit. This causes a short-circuit.

A short-circuit is when a wire by-passes a device in a circuit.



Short-circuiting a device just by-passes it: it stays off. It is easier for the current to go thru the wire than the light bulb.

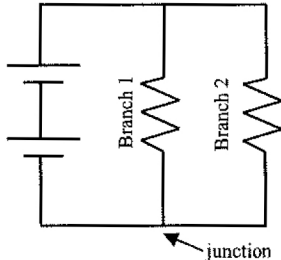


Short-circuiting a battery can be dangerous: it will drain the battery quickly and can lead to a melted wire or even a fire!

V, R, and I in Parallel Circuits

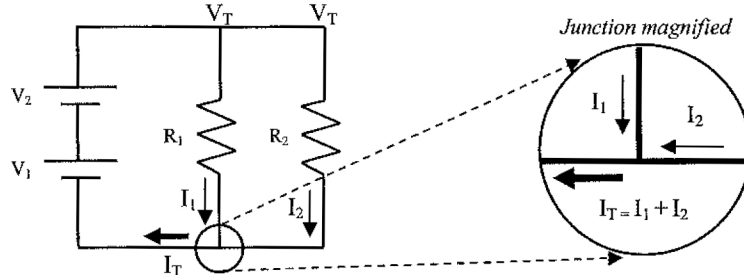
Parallel Circuits Basics

Parallel circuits have independent paths. We call these independent paths "branches".



Since wires use no voltage, we know that both branches have the same voltage.

Also, we know that all the current coming into a junction must go out.

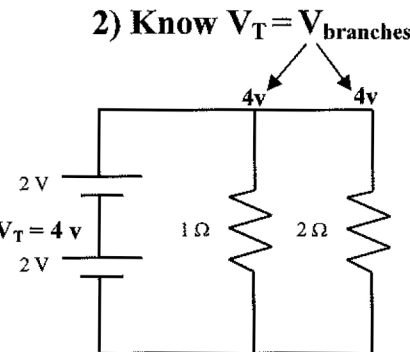


IT in a Parallel Circuit

Follow these steps to find Total Current (I_T)

1) Find V_T

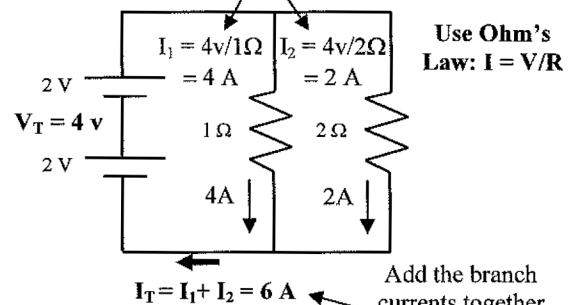
These batteries are in series, so you add them together.
 $V_T = V_1 + V_2 = 4V$



2) Know $V_T = V_{\text{branches}}$

3) Find I in each branch:

Treat each branch as its own series circuit.



Add the branch currents together to get the total current.

Going farther 5) Finding Total Resistance (R_T)

Once you know V_T and I_T , you can find R_T by Ohm's Law:
 If $V = IR$, then $R = V/I$. $R = 4v/6A = 2/3 \Omega = 0.67 \Omega$.

4) Find Total Current (I_T)

Electrical Power

Electrical Power:
 Power (in watts) $\rightarrow P = VI$
 Voltage (in volts) \leftarrow
 Current (in amps) \leftarrow
Power equals the voltage times the current.

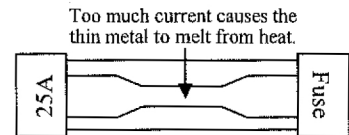
This equation gives us the same *watts* as $P = W/t$. How? First you have to know that $V = \text{Joules/Coulomb}$ and $I = \text{Coulombs/Second}$. Canceling out units gives us:

$$P = VI = \frac{\text{Joules}}{\text{Coulombs}} \times \frac{\text{Coulombs}}{\text{Second}}$$

$$= \frac{\text{Joules}}{\text{Second}} = \frac{W}{T} = \text{Power}$$

Fuses

Electricity cause heat. *Fuses melt* (or break) when too much current passes through it, protecting expensive electronic equipment. Circuit breakers protect against too much current like fuses, but can be reset.



Electrons

The electrons that move to make electricity come mostly from the wires in the circuit, not from the battery. Metals are conductors because their electrons can move.

