

ELKHATIB

This number sense booklet is supposed to aid students in:

- 1. Transitioning from elementary Math to High school and College Math.
- 2. Doing mental Math.
- 3. Becoming less fearful of Math Classes.
- 4. Giving them a chance to compete in number sense in their schools.

The best way to benefit from this study guide is to go over the directions multiple times till you master the concept, and then practice the problems at the end of each section mentally if you can. More importantly, do not proceed to the next section unless you have mastered the previous one. One great thing about this booklet is you can go over it many times and never get bored.



I dedicate this booklet to my late mother Mahassen Abou Housah. She is my heroine. She taught me how to be passionate, diligent and caring.Mom; there is no one like you. Love you.

Your loving son

Dr.Elkhatib

WELCOME

ELKHATIB

TABLE OF CONTENTS

SQUARING NUMBERS ENDING IN 5	4
MULTIPLYING EVEN DIGITS BY 2-DIGIT NUMBERS ENDING WITH 5	6
DIVIDING BY 0.01, 0.05, 0.10, 0.50	8
MULTIPLYING BY 11	10
ADDING NUMBERS IN A SERIES	12
CUBES AND SQUARES TO MEMORIZE	14
MUTIPLYING TWO NUMBERS CLOSE TO BUT LESS THAN 100	16
MUTIPLYING TWO NUMBERS CLOSE TO BUT GREATER THAN 100	18
DIFFERENCE OF TWO SQUARES	20
SQUARING NUMBERS THAT DO NOT END WITH 5	22
SOLVING PROPORTIONS MENTALLY	24
DIVIDING FRACTIONS MENTALLY	26
ADDING & SUBTRACTING FRACTIONS MENTALLY	29
SIMPLIFYING COMPLEX FRACTIONS MENTALLY	31

SQUARING NUMBERS ENDING IN 5

PROCEDURE		
Step 1	Square the 5 in the ones place. Write 25	25
Step 2	Omit the 5 from your number and multiply the rest of the number by 1 more than the itself. Write that number.	
NOTE	An exponent is a raised symbol beside the numeral indicating how many times the number should be used as a factor. So for 4 ² , the 2 is the exponent.	

EXAMPLE

Evaluate 25² 1

Step 1	Square 5, and write 25.	25
Step 2	Multiply the number 2 by 1 more than itself. $2 \times 3 = 6$	ANSWER: 625
2 Evalua	te 95 ²	
Step 1	Square 5, and write 25.	25

Step i	Square 5, and write 25.	23
Step 2	Multiply 9 by 1 more than itself. $9 \times 10 = 90$	ANSWER: 9025

NUMBER SENSE

SQUARING NUMBERS ENDING IN 5

MBER SENSE -	NUMBER SENSE NUMB	
PA	RT A	
R SENSE - NUM	BER SENSE - NUMBER SEN	101
1	5 ² =	
2	95 ² =	
3	75 ² =	
4	105 ² =	
5	65 ² =	
6	115 ² =	
7	55 ² =	
8	85 ² =	
9	35 ² =	
10	45 ² =	
MBER SENSE -	NUMBER SENSE NUMB	
PA	RT B	
R SENSE - NUMI	BER SENSE - NUMBER SEN	
1	105×105	; =
2		
	85×85	=
3	85×85 35×35	=
3 4		
3 4 5	35×35	=
3 4 5 6	35 × 35 115 × 115	=
5	35 × 35 115 × 115 15 × 15	= = =
5	35×35 115 × 115 15 × 15 95 × 95	= = =
5 6 7	35×35 115 × 115 15 × 15 95 × 95 55 × 55	= = = =

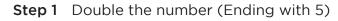
PRACTICE

10 65×65 =

MULTIPLYING EVEN DIGITS BY **2-DIGIT NUMBERS ENDING WITH 5**

PROCEDURE

NUMBER SENSE



- **Step 2** Half the other number
- **Step 3** Multiply both results

EXAMPLE

What is $25 imes 18$	2
-----------------------	---

Step 1	Double 25	50
Step 2	Half 18	9
Step 3	50×9 = 450	
2 What is	s 45×14 ?	
2 What is Step 1	5 45 × 14 ? Double 45	90

Step 3 90×7 = 630

6

ELKHATIB 7

MULTIPLYING EVEN DIGITS BY 2-DIGIT NUMBERS ENDING WITH 5

1	35×12	=
2	45×18	=
3	25×14	=
4	25×22	=
5	55×14	=
6	75×12	=
7	85×14	=
8	25×24	=
9	35×26	=
10	65×12	=

PRACTICE

DIVIDING BY 0.01, 0.05, 0.10, 0.50

PROCEDURE

NUMBER SENSE



Think of 0.01 as 1 penny

0.05 as 1 nickel

0.10 as 1 dime

0.25 as 1 quarter

0.50 as 50 cents

EXAMPLE



Divide 15 b	ıy 0.01
Procedure	Think of 15 as \$15 and 0.01 as 1 penny. Ask yourself: How many pennies are there in \$15? Since there are 100 pennies in \$1, therefore there are 1500 pennies in \$15.
ANSWER:	1500
2 Divide 12 b	y 0.05
Procedure	Think of 12 as \$12 and 0.05 as 1 nickel. Ask yourself: How many nickels are there in \$12? Since there are 20 nickels in \$1, therefore there are 240 nickels in \$12.
ANSWER:	240
3 Divide 24	by 0.10
Procedure	There are 10 dimes in \$1
ANSWER:	240
4 Divide 42	by 0.25
Procedure	There are 4 quarters in \$1
ANSWER:	42×4 = 168
5 Divide 32 I	by 0.50
Procedure	There are 2 50-cents in \$1
ANSWER:	32×2 = 64

8

DIVIDING BY 0.01, 0.05, 0.10, 0.50

1	14÷0.05	=
2	18÷0.01	=
3	24÷0.05	=
4	26÷0.50	=
5	22÷0.25	=
6	32÷0.25	=
7	82÷0.25	=
8	28÷0.25	=
9	18÷0.10	=
10	84÷0.05	=
11	124÷0.01	=
12	42÷0.05	=
13	38÷0.05	=
14	240÷0.01	=
15	180÷0.10	=
16	38÷0.50	=
17	284÷0.50	=
18	824÷0.50	=
19	428÷0.01	=
20	1820÷0.50	=

PRACTICE

NUMBER SENSE

MULTIPLYING BY 11

PROCEDURE

PROCEDURE	
Step 1	Write the digit in the ones place.
Step 2	Add the digit in the tens plcae to the digit in the ones place. Write digit in the ones place in the answer. Carry any tens.
Step 3	Move over one place to the left . Add that digit to its neighbor on the right, remembering to add any digit carried
Step 4	Continue adding each digit to its neighbor on the right, carrying if necessary.
Step 5	The digit on the far left, plus any digit carried, becomes the first digit of the answer.

 $\rangle\rangle\rangle$

EXAMPLE

1

What is 11 imes 287 ?

Step 1	Write the digit in the ones column	7	
Step 2	Add 8 + 7 = 15 Write the 5 and carry the 1.	57	
Step 3	Add 2 + 8 = 10 Plus the one that was carried is 11. Write the 1 and carry the 1.	157	
Step 4	Add the 1 to the 2, the last digit on the left, equals 3.	3,157	

2 What is $11 \times 6,657$?

Step 1	Write the digit in the ones column	7	
Step 2	Add 5 + 7 = 12 Write 2 and carry 1.	27	
Step 3	Add 6 + 5 = 11, plus the 1 carried = 12. Write the 2 and carry the 1.	227	
Step 4	Add 6 + 6 = 12 plus the 1 carried = 13. Write the 3 and carry 1	3,227	
Step 5	Add 1 to the 6, the last digit on the left, equals 7.	73,227	

MULTIPLYING BY 11

MBER	SENSE - NUMBER SENSE NUMBER	
	PART A	
RSEN	SE - NUMBER SENSE - NUMBER SENSE	
1	11×57,902	=
2	11×267,179	=
3	11×3,426,987	=
4	11×5,852,493	=
5	11×8,219,185	=
6	11×6,754,911	=
7	11×1,831,971	=
8	11×6,465,239	=
9	11×480,994,232	=
10	11×938,183,214	=

PRACTICE



ADDING NUMBERS IN A SERIES

SKILL

Step 1 Add the smallest number of the group to the largest number.

Step 2 Multiply the sum by the number of addends

Step 3 Divide the product by 2.

EXAMPLE

Add 1+2+3+4+......50

NUMBER SENSE

Add 1 to 50. You get \$51. Multiply $51 \times 50 = 2550$. **Procedure:** Divide the result by 2 $2550 \div 2 = 1275$

2 What is the sum of 44 + 48 + 52 + 56?

Procedure:

Add 44+56=100 Add 48+52=100 Add the 2 results: 100+100=200

 $\mathbf{3}$ What is the sum of 32 + 33 + 34 + 35 ? (This method works for consecutive numbers.)

 Multiply 30 by 4, you get 120.

 Procedure:
 Now add 2+3+4+5=14

 Add the 2 results 120+14=134

PRACTICE

ADDING NUMBERS IN A SERIES

>	Solve:
1	1+2+3+4++60=
2	41+45+42+43=
3	1+2+3+4+5++100=
4	23+22+21+27=
5	36+32+64+68=
6	1+2+3+4+5+6++40=
7	62+61+63+65=
8	51+52+53+55=
9	21+32+29+48=

NUMBER SENSE

CUBES AND SQUARES TO MEMORIZE

SQUARES

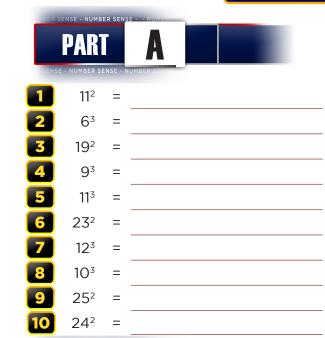
CUBES

9 ²	81
10 ²	100
11 ²	121
12 ²	144
13 ²	169
14 ²	196
15 ²	225
16 ²	256
17 ²	289
18 ²	324
19 ²	361
20 ²	400
21 ²	441
22 ²	484
23 ²	529
24 ²	576
25 ²	525

1 ³	1
2 ³	8
3 ³	27
4 ³	64
5 ³	125
6 ³	216
7 ³	343
8 ³	512
9 ³	729
10 ³	1,000
11 ³	1331
12 ³	1728

CUBES AND SQUARES TO MEMORIZE

PRACTICE





- NUMBER SENSE

1	25 ²	=	
2	12 ³	=	
3	10 ³	=	
4	8 ³	=	
5	23 ²	=	
6	7 ³	=	
7	22 ²	=	
8	21 ²	=	
9	18 ²	=	
10	5 ³	=	

MUTIPLYING TWO NUMBERS CLOSE TO BUT LESS THAN 100

PROCEDURE

NUMBER SENSE



- Step 1Find the difference of each of the numbers from 100.Step 2Multiply these differences; write the product as the last two digits of the
answer on the right.Subtract the difference of one number from the other original number; write
- **Step 3** Subtract the difference of one number from the other original number, which the difference as the first two digits of the answer on the left.

EXAMPLE



What is 94 imes 97 ?

Step 1	Find the difference of 94 and 97 from 100	6;3
Step 2	Multiply 6 and 3; write 18 as the last two digits of the answer on the right	18
Step 3	Subtract the difference of one number from the other original number; 94 - 3 = 91 or 97 - 6 = 91; write 91 as the first two digits of the answer on the left.	9118

2 What is 92×91 ?

Step 1	100 - 92 = 8;100 - 91 = 9	8; 9
Step 2	8×9 = 72	72
Step 3	92-9 = 83 or 91-8 = 83	8372

MULTIPLYING TWO NUMBERS CLOSE TO BUT LESS THAN 100



PRACTICE



SENSE - NUMBER SENSE - NUMBER S

1	89×95	=	
2	96×93	=	
3	99×91	=	
4	94×96	=	
5	88×98	=	
6	99×89	=	
7	87×97	=	
8	96×90	=	
9	89×91	=	
10	90×94	=	

MUTIPLYING TWO NUMBERS CLOSE TO BUT GREATER THAN 100

PROCEDURE

NUMBER SENSE



- Step 1Find the difference of each of the numbers from 100.Step 2Multiply these differences; write the product as the last two digits of the answer
on the right.Add the differences of one number to the other original numbers write the sum
- Step 3Add the differerence of one number to the other original number; write the sum
as the first three digits of the answer on the left.

EXAMPLE

1



What is 104 imes 102 ?

Step 1	Find the difference of 104 and 102 from 100.	4; 2
Step 2	Multiply 4 and 2; write 08 as the last two digits of the answer on the right.	08
Step 3	Add the difference of one number to the other original number; 4 + 102 = 106 or 2 + 104 = 106; write the sum (106) as the first three digits of the answer on the left.	10608

$\mathbf{2}$ What is 108 imes 111 ?

Step 1	108 - 100 = 8; 111 - 100 = 11	8; 11
Step 2	8×11 = 88	88
Step 3	11 + 108 = 119 or 8 + 111 = 119	11988

ELKHATIB 19

MULTIPLYING TWO NUMBERS CLOSE TO BUT GREATER THAN 100

MBER	SENSE - NUMBER SENSE	NU
	PART	4
R SEN	SE - NUMBER SENSE - NU	MBER SE
1	102×109	=
2	103× 107	=
3	106× 102	=
4	109× 105	=
5	102× 108	=
6	101×110	=
7	111×106	=
8	109×104	=
9	105× 106	=
10	112×103	=
MBER	SENSE - NUMBER SENSE	NUN
MBER	PART	B
MBER SEN	PART SE - NUMBER SENSE - NU	MBER SI
	PART SE - NUMBER SENSE - NU 104×103	MBER SI
SEN 1	PART SE - NUMBER SENSE - NU 104×103 108×104	MBER SI
1 2 3	PART SE - NUMBER SENSE - NU 104×103 108×104 113×103	Mber si = =
1 2 3 4	PART SE - NUMBER SENSE - NU 104×103 108×104 113×103 105×102	MBER SI = = =
1 2 3 4 5	PART 104×103 108×104 113×103 105×102 103×108	MBER ST = = = =
1 2 3	PART 104×103 108×104 113×103 105×102 103×108 110×109	MBER SI = = = = =
1 2 3 4 5 6 7	PART 104×103 108×104 113×103 105×102 103×108 110×109 107×108	
1 2 3 4 5	PART 104×103 108×104 113×103 105×102 103×108 110×109 107×108 104×106	
1 2 3 4 5 6 7	PART 104×103 108×104 113×103 105×102 103×108 110×109 107×108	

PRACTICE

NUMBER SENSE

SQUARES

PROCEDURE

- $a^{2} b^{2} = (a + b)(a b)$ Step 1 Find the sum of the numbers that are being squared.
- Find the difference of the numbers that are being squared. Step 2
- Find the product of the sum and difference of the numbers being squared. Step 3

DIFFERENCE OF TWO

EXAMPLE

What is 13² – 9² ?

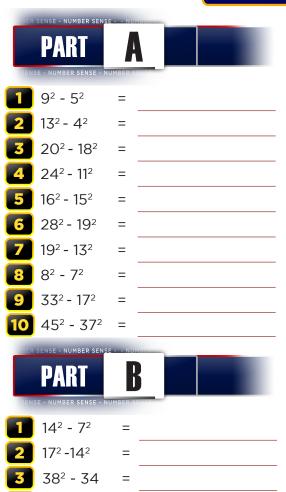
Step 1	Find the sum of 13 and 9. 13 + 9 = 22	22
Step 2	Find the difference of 13 and 19. 13 – 9 = 4	4
Step 3	Find the product of 22 and 4. $22 \times 4 = 88$	88

2 What is 27² – 18² =

Step 1	Find the sum of 27 and 18. 27 + 18 = 45	45
Step 2	Find the difference of 27 and 18. 27 – 18 = 9	9
Step 3	Find the product of 45 and 9. $45 \times 9 = 405$	405

DIFFERENCE OF TWO SQUARES





29² - 25²

14²- 12²

44²-24²

82² - 78²

48² - 43²

42² - 36²

98² - 89²

=

=

=

=

=

=

=

4

5

6

7

8

9

10

SQUARING NUMBERS THAT DO NOT END WITH 5

RULE

 $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$

EXAMPLE



Evaluate: 23²

NUMBER SENSE

SOLUTION

- **Step 1** Rewrite 23² as (20 + 3)²
- Step 2 Square 20. You get 400.
- Step 3Multiply 2×20×3, you get 120
- Square 3, you get 9. Finally add all the results. 400 + 120 + 3 = 523.
- Step 4 ANSWER: 523

2 Evaluate: 37²

SOLUTION

- **Step 1** Rewrite 37² as (40-3)²
- Step 2 Square 40. You get 1600.
- **Step 3** Multiply $2 \times 40 \times 3$, you get 240

Square 3, you get 9. Finally subtract 1600 - 240 and add 9 to the result.

Step 4 1360 + 9 = 1369 **ANSWER:** 1369

SQUARING NUMBERS THAT DO NOT END WITH 5

		_	
INSTR	υςτις	ONS	Evaluate
1	83 ²	=	
2	62 ²	=	
3	54²	=	
4	91 ²	=	
5	81²	=	
6	21 ²	=	
7	32 ²	=	
8	51 ²	=	
9	7 1 ²	=	
10	88²	=	
11	98²	=	
12	39 ²	=	
13	48 ²	=	
14	69 ²	=	
15	89²	=	

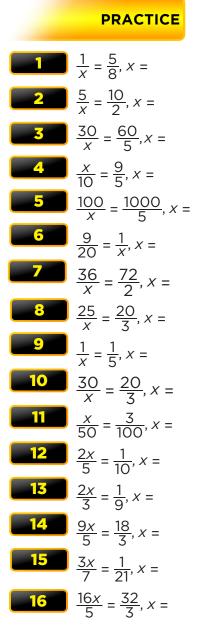
PRACTICE

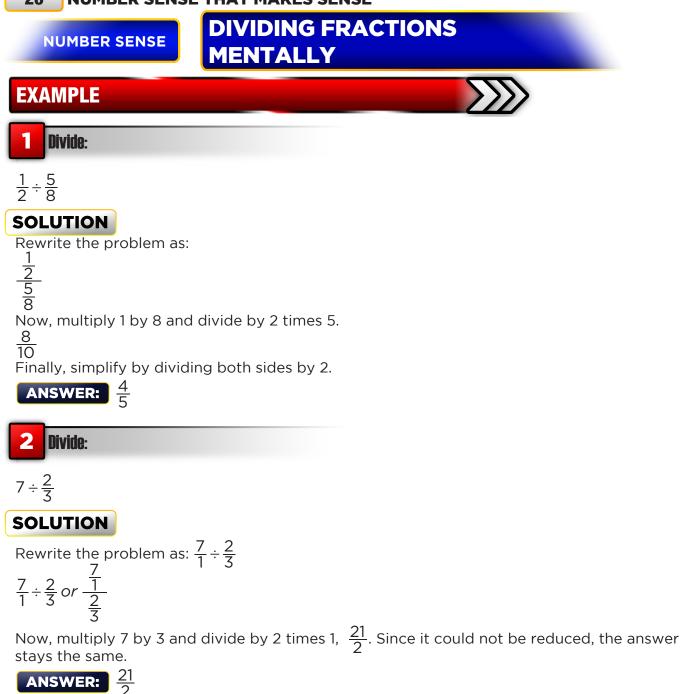
NUMBER SENSE

SOLVING PROPORTIONS MENTALLY

1 If
$$\frac{a}{b} = \frac{c}{d}$$
, then $\frac{b}{a} = \frac{d}{c}$.
EXAMPLE: $\frac{1}{x} = \frac{5}{6}$, then $\frac{x}{1} = \frac{6}{5}$ or $x = \frac{6}{5}$.
2 If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.
EXAMPLE: $\frac{10}{x} = 5$, then $\frac{10}{5} = \frac{x}{1}$ or $x = 2$.
3 If $\frac{a}{b} = \frac{c}{d}$, you can reduce a and c or b and d .
EXAMPLE: $\frac{15}{x} = \frac{5}{3}$, divide 15 and 5 by 5, you get: $\frac{3}{x} = \frac{1}{3}$, cross multiply: $x = 9$
EXAMPLE: $\frac{x}{75} = \frac{1}{25}$, divide 75 and 25 by 25, you get: $\frac{x}{3} = \frac{1}{1}$, cross multiply: $x = 3$

SOLVING PROPORTIONS MENTALLY





DIVIDING FRACTIONS MENTALLY

CONTINUED



 $\frac{8}{9} \div 7$

SOLUTION

Rewrite the problem as:

$$\frac{8}{9} \div \frac{7}{1} \text{ or } \frac{\frac{8}{9}}{\frac{7}{1}}$$

Now, multiply 8 by 1 and divide by 9 times 7.

<u>8</u> 63

Since it could not be reduced, the answer stays the same.



4 Divide:

$$2\frac{1}{3} \div 5\frac{1}{4}$$

SOLUTION

Convert each fraction into an improper fraction:

 $2\frac{1}{3} becomes \frac{7}{3}$ $5\frac{1}{4} becomes \frac{21}{4}$

Now, rewrite the problem as:

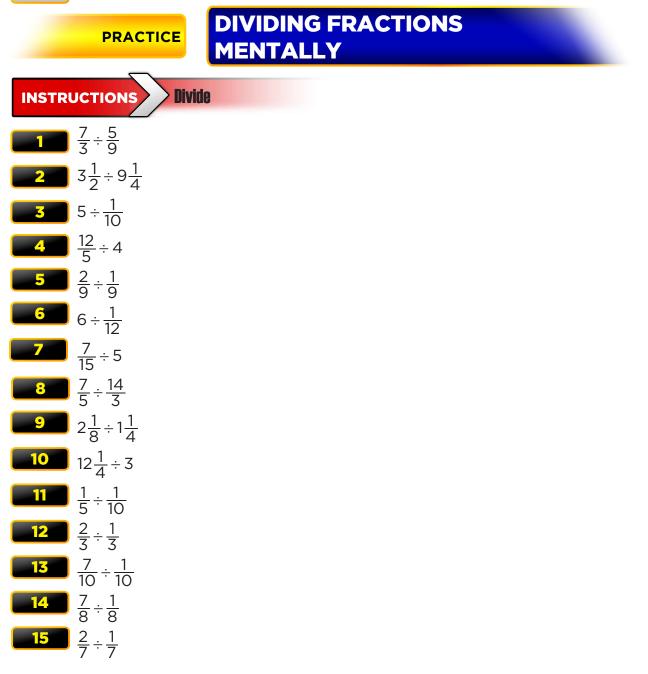


Multiply 7 by 4 and divide by 3 times 21.

<u>28</u> 63

Divide both sides by 7.





ADDING & SUBTRACTING NUMBER SENSE **FRACTIONS MENTALLY**

BACKGROUND INFORMATION

If you were to add or subtract fractions where the denominators do not have a GCF. The butterfly method is the quickest way to perform such operations.

EXAMPLE





SOLUTION

- Multiply the denominators first. $5 \times 6 = 30$ ٠
- Now cross multiply from left to right and add. $3 \times 6 + 1 \times 5 = 18 + 5 = 23$.

ANSWER:	23
ANSWER:	30

Subtract:



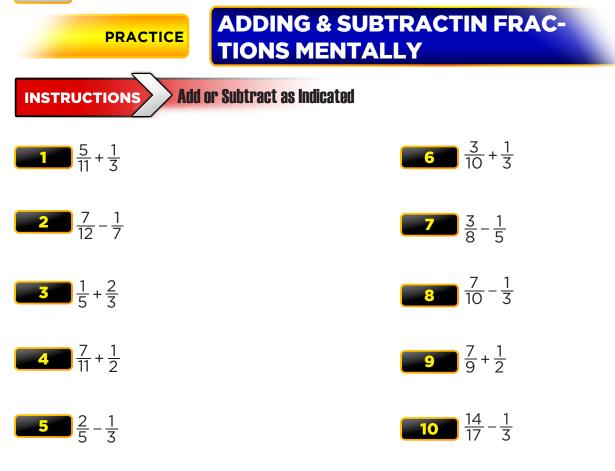
SOLUTION

- Multiply the denominators first. $7 \times 3 = 21$ ٠
- Now cross multiply from left to right and subtract. $5 \times 3 1 \times 7 = 8$ ٠









NUMBER SENSE

EXAMPLE

Simplify:

<u>1</u> 2	+	<u>1</u> 3
<u>1</u> 6	+	<u>1</u> 2

SOLUTION

Find the L.C.D. of all 4 denominators namely: 2, 3, 6, 2. The answer is 6. Now, multiply 6 by each fraction. $(6)\frac{1}{2} + (6)\frac{1}{3}$ $(6)\frac{1}{6} + (6)\frac{1}{2}$

SIMPLIFYING COMPLEX

FRACTIONS MENTALLY

Simplify: $\frac{3+2}{1+3} = \frac{5}{4}$

ANSWER: $\frac{5}{4}$

2 Simplify:

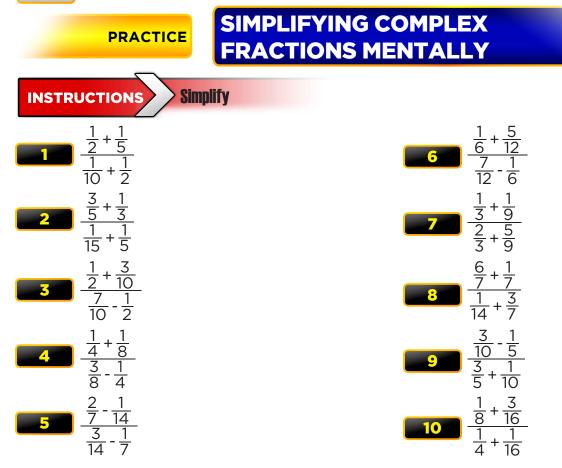
<u>6</u> 7	$-\frac{1}{3}$
$\frac{4}{7}$	$+\frac{1}{3}$

SOLUTION

Find the L.C.D. of all 4 denominators namely: 7, 3, 7, 3. The answer is 21. Now, multiply 21 by each fraction. $\frac{(21)\frac{6}{7} - (21)\frac{1}{3}}{(21)\frac{4}{7} + (21)\frac{1}{3}}$

Simplify: $\frac{18 - 7}{12 + 7} = \frac{11}{19}$





YOU HAVE REACHED THE END OF THIS INSTRUCTIONAL MATERIAL



END

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