Math 0312
~ Review For Final ~

A,

Graph:

1,

$-4x - 10y \leq 10$

~ solution ~

Solve for y first.

$-10y \leq 4x + 10$

Multiply both sides of the inequality by -1.

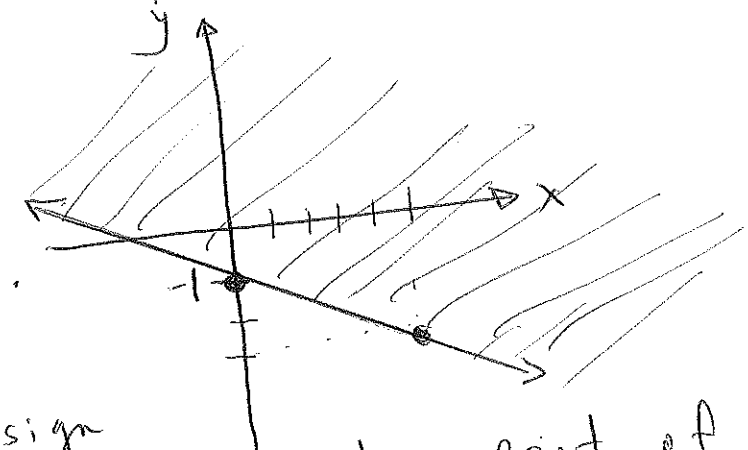
$10y \geq -4x - 10$

Divide each term by 10.

$y \geq \frac{-2}{5}x - 1$

Graph the line $y = \frac{-2}{5}x - 1$

$b = -1$. slope $= \frac{-2}{5} = \frac{\text{Rise}}{\text{Run}}$.



Since the sign is $\geq$ shade the top part of the line.

2,

$2x - 5y > 5$

- solution -

Solve for y first.

$-5y > -2x + 5$

Multiply by -1.

$5y < 2x - 5$

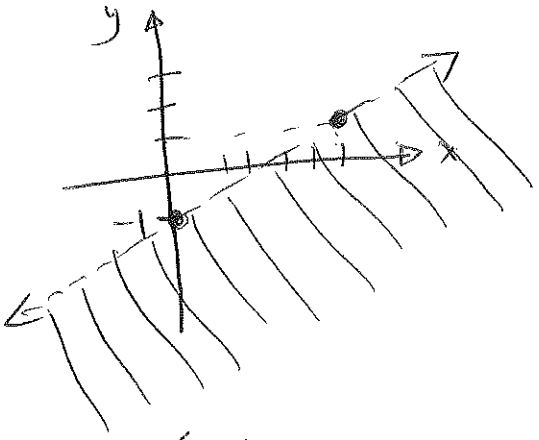
Divide by 5.

$y < \frac{2}{5}x - 1$ (Detached line)

Graph: $y = \frac{2}{5}x - 1$

$b = -1$ $\quad m = \frac{2}{5} \leftarrow$ Rise / Run

$<$ $\Rightarrow$ shade under the line.



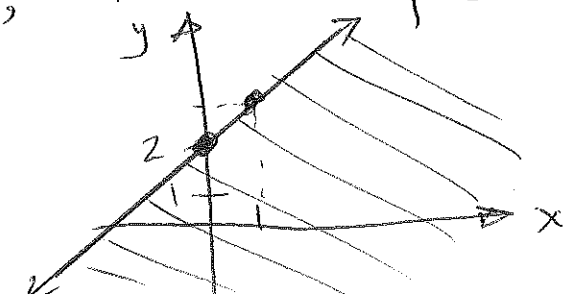
3, $-x + y \leq 2$

- Solution -

Solve for y first.

$y \leq x + 2$

$b = 2$; $m = 1$ or $\frac{1}{1} \leftarrow$ Rise / Run



B, Multiply:

1, $(x-5)(x^2+3x+2)$

- Solution -

First multiply x by x^2+3x+2

$$x^3+3x^2+2x$$

Now multiply -5 by x^2+3x+2

$$-5x^2-15x-10$$

Now Combine both results:

$$x^3+3x^2+2x+-5x^2-15x-10.$$

Combine Like Terms: $x^3-2x^2-13x-10$

2, $(x+4)(x^2-3x-2)$

- Solution -

First multiply x by x^2

$$= x^3-3x^2-2x$$

Multiply $+4$ by x^2-3x-2

$$+4x^2-12x-8$$

Combine both results:

$$x^3-3x^2-2x+4x^2-12x-8$$

$$= x^3+x^2-14x-8$$

3,

$(x-8)(x^2+3x+7)$

- Solution -

First multiply x by x^2+3x+7

x^3+3x^2+7x

Now multiply -8 by x^2+3x+7

$-8x^2-24x-56$

Now combine both results:

$x^3+3x^2+7x+-8x^2-24x-56$

$= x^3-5x^2-17x-56$

C, Solve:

1, $2x^2+10x=-12$

- Solution -

It is a Quadratic equation.

Set the equation to 0.

$2x^2+10x+12=0$.

Divide each term by 2.

$x^2+5x+6=0$

Factor:

$(x+2)(x+3)=0$

By setting each factor to 0.

$x=-2$ or $x=-3$

2, $x^2-x-6=0$

- Solution -

Factor: product $=-6$; Sum $=-1$. Numbers are: -3 and $+2$.

$(x-3)(x+2)=0$

$x=3$ or $x=-2$.

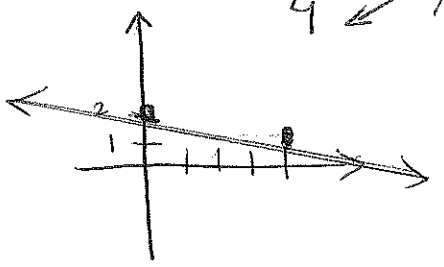
D, Graph:

1,

$y = -\frac{1}{4}x + 2$

- solution -

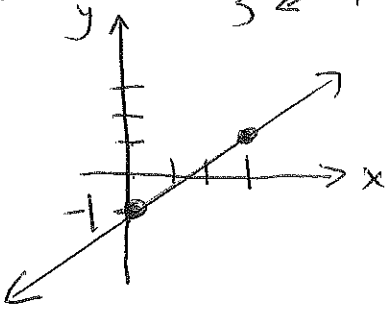
$b = 2$, $m = \frac{-1}{4}$ ← Rise / Run



2, $y = \frac{2}{3}x - 1$

- solution -

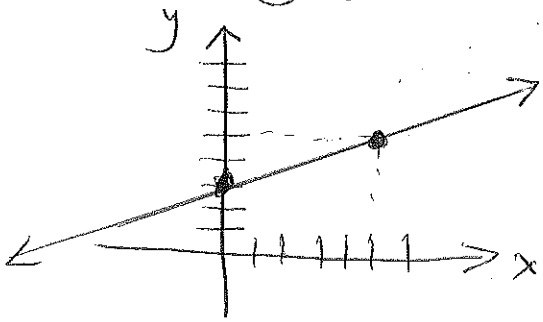
$b = -1$; $m = \frac{2}{3}$ ← Rise / Run



3, $y = \frac{2}{5}x + 3$

- Solution -

$b = 3$; $m = \frac{2}{5}$ ← Rise / Run



E, Solve:

1,

$-2x + 1 < 5$

- Solution -

$-2x + 1 < 5$

$-1 \quad -1$

$-2x < 4$

Divide both sides by -2

$\frac{-2}{-2}x < \frac{4}{-2}$

$x > -2$

2,

$-5x - 3 > 7$

- Solution -

$-5x - 3 > 7$

$+3 \quad +3$

$-5x > 10$

Divide both sides by -5

$x < -2$

3,

$-9x - 7 \leq 2$

- Solution -

$-9x - 7 \leq 2$

$+7 \quad +7$

$-9x \leq 9$ Divide both sides by -9

$x \geq -1$

F,

Perform the indicated operation and simplify

1/

$$\frac{x^2+2x}{x+1} + \frac{1}{x+1}$$

- solution -

Since both terms (fractions) have the same denominator, just add the numerators:

$$\frac{x^2+2x+1}{x+1}$$. Now factor the numerator:

$$\frac{(x+1)(x+1)}{(x+1)}$$. Crossing out $(x+1)$ from both the numerator & denominator.

$$= x+1$$

2, $$\frac{x-5}{x+7} - \frac{x}{x+7}$$

- Solution -

Since both fractions have the same denominator; subtract the numerators:

$$\frac{x-5-x}{x+7} = \frac{-5}{x+7}$$

3, $$\frac{x^2+5x}{x-6} + \frac{6}{x-6}$$

- Solution -

Since both fractions have the same denominator, just add the numerators:

$$\frac{x^2+5x+6}{x-6}$$. Factor the numerator: $$\frac{(x+2)(x+3)}{x-6}$$

G, Solve:

1/

$x+y=5$

$x-y=3$

- Solution -

Add both equations:

$2x+0=8$

$2x=8 \rightarrow x=4.$

Replace $x=4$ in $x+y=5$

$4+y=5$

$-4 \quad -4$

and $y=1$. Answer is $(4, 1)$

2, $2x+y=0$

$x-y=3$

- Solution -

Add both Equations:

$3x=3$

Divide both sides by 3:

$x=1.$

Substitute $x=1$ in $2x+y=0$

$2(1)+y=0$

$2+y=0$

$-2 \quad -2$

$y=-2$. Answer is $(1, -2)$

3,

$$8x + y = 5$$
$$-8x + 3y = 3$$

- Solution -

Add both Equations:

$$4y = 8 \Rightarrow y = 2.$$

Replace $y = 2$ in $8x + y = 5$

$$8x + 2 = 5$$
$$-2 \quad -2$$
$$8x = 3 \Rightarrow x = \frac{3}{8}$$

$$\left(\frac{3}{8}, 2\right)$$

H, Simplify:

1, $$\frac{\frac{2}{x} + 1}{\frac{2}{x} - 1}$$

- Solution -

This is a complex fraction. Multiply each term by x.

$$\frac{x \cdot \frac{2}{x} + x(1)}{x \cdot \frac{2}{x} - 1(x)} = \frac{2 + x}{2 - x}$$

2, $$\frac{\frac{7}{x^2} + 1}{\frac{7}{x^2} - 1}$$

- Solution -

Multiply each term by x^2.

$$\frac{x^2 \cdot \frac{7}{x^2} + x^2(1)}{x^2 \cdot \frac{7}{x^2} - x^2(1)} = \frac{7+x^2}{7-x^2}$$

I, Find the slope and y-intercept:

1, $x + 5y = 1$

- Solution -

Solve for y:

$5y = 1 - x$

Divide each term by 5:

$y = \frac{1}{5} - \frac{x}{5}$

$b = \frac{1}{5}$, $m = \frac{-1}{5}$.

2, $3x - y = 7$

- Solution -

Solve for y:

$-y = 7 - 3x$

Multiply by -1.

$y = -7 + 3x \Longrightarrow b = -7,\ m = 3$.

3, $2x + 3y = 0$

- Solution -

Solve for y:

$3y = -2x$

Divide both sides by 3:

$y = -\frac{2}{3}x \Longrightarrow b = 0,\ m = \frac{-2}{3}$

J, Solve:

1/

$3m - 5 + 3(m-1) = -2m + 7$

~ Solution ~

Distribute:

$3m - 5 + 3m - 3 = -2m + 7$

Combine Like Terms:

$6m - 8 = -2m + 7$

Add $2m$ to both sides of the equation:

$8m - 8 = 7$

Add 8 to both sides of the equation:

$8m = 15$ and $m = 15/7$

2/ Solve: $8m - 2 + 2(m-5) = -3m + 1$

~ Solution ~

Distribute:

$8m - 2 + 2m - 10 = -3m + 1$

Combine Like Terms:

$10m - 12 = -3m + 1$

Add $3m$ to both sides of the equation:

$13m - 12 = 1$. Add 12 to both sides of the equations:

$13m = 13$ and $m = 1$

3, Solve:

$7(m-2) - 5m = m + 1$

- Solution -

Distribute: $7m - 14 - 5m = m + 1$

Combine Like Terms:

$2m - 14 = m + 1$

Subtract m from both sides of the equation.

$m - 14 = 1$. Add 14 to both sides of the equation.

$m = 15$

K, Factor Completely:

1/

$ax + bx + ay + by$

- Solution -

This is the grouping type:

$(ax + bx) + (ay + by)$

Factor each term:

$x(a+b) + y(a+b)$.

Take $(a+b)$ as a Common factor:

$(a+b)(x+y)$.

2,

$81x^2 + 90x + 25$

- Solution -

$\sqrt{81x^2} = 9x$

$\sqrt{25} = 5$.

Try: $(9x+5)(9x+5)$ as an answer.

check it by multiplying:

$81x^2 + 45x + 45x + 25$

$= 81x^2 + 90x + 25$. It checks.

3, $3x - 21 - x + 7$

- Solution -

Group the terms:

$(3x - 21) - (x - 7)$

Take a common factor of each term:

$3(x-7) - 1(x-7)$

Take $(x-7)$ as a common factor:

$(x-7)(3-1) = (x-7)2$

4) Simplify:

1) $64^{-4/3}$

- Solution -

$$64^{-4/3} = \frac{1}{64^{4/3}} = \frac{1}{\sqrt[3]{64^4}}$$

$$= \frac{1}{\sqrt[3]{64 \times 64 \times 64 \times 64}} \quad \text{But } \sqrt[3]{64} = 4.$$

$$= \frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}$$

2) $125^{-4/3}$

- Solution -

$$125^{-4/3} = \frac{1}{125^{4/3}} = \frac{1}{\sqrt[3]{125^4}}$$

$$= \frac{1}{\sqrt[3]{125 \times 125 \times 125 \times 125}} \quad \text{But } \sqrt[3]{125} = 5$$

$$= \frac{1}{5 \times 5 \times 5 \times 5} = \frac{1}{625}$$

M, Simplify:

1,

$\sqrt{18} + 7\sqrt{2} - \sqrt{5}$

- Solution -

We can not Combine radicals if they are of different family:

$\sqrt{9 \times 2} + 7\sqrt{2} - \sqrt{5}$

$= 3\sqrt{2} + 7\sqrt{2} - \sqrt{5}$

Combine Like Terms:

$10\sqrt{2} - \sqrt{5}$

2, $\sqrt{125} - \sqrt{50} + 2\sqrt{5}$

- Solution -

Rewrite the problem as:

$\sqrt{25 \times 5} - \sqrt{25 \times 2} + 2\sqrt{5}$

$= 5\sqrt{5} - 5\sqrt{2} + 2\sqrt{5}$

Combine Like Terms:

$7\sqrt{5} - 5\sqrt{2}$

3, $3\sqrt{2} - \sqrt{18} + \sqrt{128}$

- Solution -

Rewrite the problem as: $3\sqrt{2} - \sqrt{9 \times 2} + \sqrt{64 \times 2}$

$= 3\sqrt{2} - 3\sqrt{2} + 8\sqrt{2} = 8\sqrt{2}$.

N, Evaluate:

1, $\left(\frac{-2}{3}\right)^3$

~ solution ~

$\left(\frac{-2}{3}\right)^3 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \frac{-8}{27}$

2, $\left(\frac{2}{5}\right)^3$

- solution -

$\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$.

3, $\left(\frac{-5}{2}\right)^3$

~ solution ~

$\left(\frac{-5}{2}\right)^3 = \frac{-5}{2} \times \frac{-5}{2} \times \frac{-5}{2} = \frac{-125}{8}$

. .

Q, Multiply:

1) $\frac{2x-2}{x} \cdot \frac{3x^2}{4x-4}$

- Solution -

Factor:

$\frac{2(x-1)}{x} \cdot \frac{3x^2}{4(x-1)}$

Cross out $(x-1)$ term:

$\frac{2}{x} \cdot \frac{3x^2}{4} = \frac{6x^2}{4x} = \frac{3}{2}x$

2) $\frac{9x-18}{x} \cdot \frac{2x^2}{x-2}$

- Solution -

Factor:

$\frac{9(x-2)}{x} \cdot \frac{2x^2}{(x-2)}$ Cross out $x-2$

$\frac{9}{x} \cdot 2x^2 = \frac{18x^2}{x} = 18x$

P, Factor:

1/ $x^2 - 7x + 12$

- Solution -

Product = +12

Sum = -7

Numbers are -4 & -3

$(x-4)(x-3)$

2, $x^2 + x - 6$

- Solution -

Product = -6

Sum = +1

Numbers are: +3 & -2

$(x+3)(x-2)$

3/ $x^2 + 2x - 35$

- Solution -

Product = -35

Sum = +2

Numbers are: +7 & -5

$(x+7)(x-5)$

Q, Divide:

1,

$$\frac{5+3i}{2+7i}$$

– Solution –

Conjugate of $2+7i = 2-7i$

Multiply by $2-7i$.

$$\frac{(5+3i)}{(2+7i)} \cdot \frac{(2-7i)}{(2-7i)}$$

Multiply: $(5+3i)(2-7i)$

$= 10 + 6i - 35i - 21i^2$

but $i^2 = -1$.

$= 10 + 6i - 35i + 21 = 31 - 29i$.

Multiply: $(2+7i)(2-7i)$

$= 4 - 49i^2$. But $i^2 = -1$

$= 4 + 49 = 53$.

Answer is: $\frac{31-29i}{53}$

2,

$$\frac{7+4i}{2+3i}$$

- Solution -

Conjugate of $2+3i$ is $2-3i$.

$$\frac{(7+4i)}{(2+3i)} \cdot \frac{(2-3i)}{(2-3i)}$$

Multiply: $(7+4i)(2-3i)$

$= 14 + 8i - 21i - 12i^2$

But $i^2 = -1$

$= 14 + 8i - 21i + 12$

$= 26 - 13i$.

Multiply: $(2+3i)(2-3i)$

$= 4 - 9i^2$. But $i^2 = -1$

$= 4 + 9 = 13$.

Divide Both answers:

$$\frac{26-13i}{13}$$ Divide each term by 13.

$$\boxed{2-i}$$

R, Solve:

1/

$|x+2| = 5$. Set the equation = 5 and then -5.

- Solution -

$x + 2 = 5$

$-2 \quad -2$

$x = 3$ ✓

$x + 2 = -5$

$-2 \quad -2$

$x = -7$ ✓

2/

$|3x - 1| = 8$

- Solution -

$3x - 1 = 8$

$+1 \quad +1$

$3x = 9$ ∴ $x = 3$ ✓

$3x - 1 = -8$

$+1 \quad +1$

$3x = -7$

∴ $x = -7/3$ ✓

5, Solve:

1,

$$3 + \frac{2}{a} = \frac{8}{a}$$

- Solution -

This is a rational equation. Multiply each term by a.

$$a(3) + a \cdot \frac{2}{a} = a \cdot \frac{8}{a}$$

$$3a + 2 = 8$$

$$-2 \quad -2$$

$$3a = 6 \quad \therefore a = 2$$

2,

$$7 + \frac{5}{a} = \frac{7}{a}$$

- Solution -

Multiply each term by a.

$$(a) \cdot 7 + a \cdot \frac{5}{a} = a \cdot \frac{7}{a}$$

$$7a + 5 = 7$$

$$-5 \quad -5$$

$$7a = 2 \quad \therefore a = \frac{2}{7}$$

T, Divide:

1, $\frac{2\sqrt{5}}{\sqrt{2}}$

- Solution -

Multiply the numerator and denominator by $\sqrt{2}$.

$$\frac{2\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}}{2} = \sqrt{10}.$$

2, $\frac{7\sqrt{5}}{\sqrt{3}}$

- Solution -

Multiply the numerator and denominator by $\sqrt{3}$.

$$\frac{7\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{15}}{3}$$

3, $\frac{8\sqrt{2}}{\sqrt{3}}$

- Solution -

Multiply the numerator and denominator by $\sqrt{3}$.

$$\frac{8\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{6}}{3}$$

IV, $f(x) = 2x^2 - 3x + 1$

find:

1, $f(-3)$

- Solution -

Replace x with -3

$2(-3)^2 - 3(-3) + 1$

$= 2(9) + 9 + 1$

$= 18 + 9 + 1 = \boxed{28}$

2, $f\left(\frac{1}{2}\right)$

- solution -

$2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$

$= 2\left(\frac{1}{4}\right) - \frac{3}{2} + 1$

$= \frac{2}{4} - \frac{3}{2} + 1$

$= 0.5 - 1.5 + 1 = 0$

V, Divide.

1, $\dfrac{6x^4 - 15x^3 + 12x^2}{3x^3}$

- Solution -

Divide each term by $3x^3$.

$\frac{6x^4}{3x^3} - \frac{15x^3}{3x^3} + \frac{12x^2}{3x^3} = 2x - 5 + \frac{4}{x}$

W, Solve the formula for the specified variable:

1, $A = P(1+nr)$ for r

- Solution -

Distibute:

$A = P + Pnr$

Solve for Pnr first

$A - P = Pnr$. Divide both sides by Pn

$\frac{A-P}{Pn} = r$

2, $F = \frac{9}{5}C + 32$ for C

- Solution -

Multiply each term by 5

$5F = 5 \cdot \frac{9}{5}C + 5(32)$

$5F = 9C + 180$

solve for C

$9C = 5F - 180$

Divide both sides by 9.

$C = \frac{5F - 180}{9}$

X) Determine the domain of the functions:

1) $f(x) = \dfrac{x^2 - 4}{x^2 + 2x - 48}$

- Solution -

The denominator can not be 0.

$x^2 + 2x - 48 = 0$

Factor: $(x+8)(x-6) = 0$

$x = -8$ or $x = 6$.

Domain will be any x except -8 or 6.

2) $f(x) = \dfrac{7x - 1}{25x^2 - 81}$

- Solution -

Set the denominator to 0.

$25x^2 - 81 = 0$

Factor: $(5x + 9)(5x - 9) = 0$

$\therefore$ When $5x + 9 = 0$

$-9 \quad -9$

$5x = -9$

$x = -9/5$.

When $5x - 9 = 0$

$+9 \quad +9$

$5x = 9$

$x = 9/5$

Domain will be any x except $-9/5$ + $9/5$

4. Simplify:

1) $(-2x^2)(7x^3)$

- Solution -

Multiply the numbers first: $-2 \times 7 = -14$.

Multiply the Variables: $x^2 \cdot x^3 = x^5$

Answer is $-14x^5$

2) $(7x^2y)(-3xy^2)$

- Solution -

Multiply the numbers first: $7 \times -3 = -21$

Now $x^2 \cdot x = x^3$

$y \cdot y^2 = y^3$

Answer is $-21x^3y^3$

2, Find the slope of the lines going through the following points:

1, Through $(-1, 5)$, $(-5, -7)$

- Solution -

$(-1, 5)$, $(-5, -7)$

x_1 y_1 x_2 y_2

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-5}{-5-(-1)} = \frac{-12}{-5+1}$$

$$= \frac{-12}{-4} = 3$$

2, Through $(3, 8)$, $(0, -5)$

- Solution -

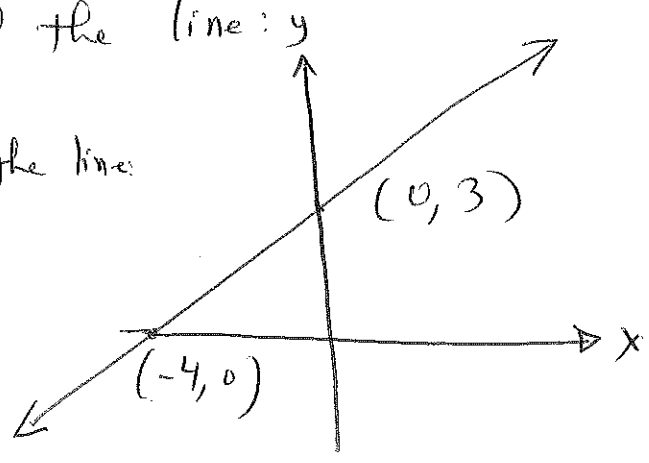
$(3, 8)$, $(0, -5)$

x_1 y_1 x_2 y_2

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-8}{0-3}$$

$$= \frac{-13}{-3} = \frac{13}{3}$$

3, Find the slope of the line:



- Solution -

Pick any 2 points on the line:

$(0, 3)$, $(-4, 0)$

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{-4-0} = \frac{-3}{-4}$$

$$= \frac{3}{4}$$